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Therefore, finally,

$$\frac{x}{\dot{x}_0} = \frac{T}{2\pi} \cdot \frac{2\sin\theta/2}{\tan\alpha} \times \text{correction for damping}$$

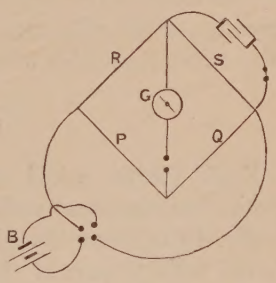
$$= \frac{CS^2}{\Delta} \cdot \frac{(PB+PR+QR+BR)(PQ+GQ+RQ+PG)}{P \,.\, dS},$$

which on expansion of Δ gives $\frac{CS^2}{dS}$;

$$\therefore \quad C = \frac{T}{2\pi} \cdot \frac{2\sin\theta/2}{\tan\alpha} \cdot \frac{dS}{S^2} \times \text{correction for damping.}$$

In the practice of the method, in order to avoid error arising from the altered value of Δ due to the change of S, readings of the deflexion, α may be taken with equal positive and negative values of dS. Thus an error which otherwise might be as great as 1 in 100 is reduced to 1 in 10^4.

In order to avoid change of the E.M.F of the battery due to its being for any appreciable time on open circuit, it is desirable to use a reversing-key in the battery branch, and to observe the throw of the galvanometer due to reversal of the sign of the current in S; and immediately after observe the deflexion due to the alteration of S.



The method is necessarily subject to the same disadvantages as that of Fleeming Jenkin, in respect of it not giving the instantaneous capacity of the condenser. But measurements of the same condenser taken with different galvanometers, or with a graded moment of inertia of the galvanometer-needle, have led to nearly constant results, the capacity being apparently but little dependent on the period of charging and discharging of the condenser. The effect would be likely to be much more marked if the capacity were much greater.

The conditions for a maximum throw of the galvanometer for a given E.M.F. are, moreover, not the same as those for the maximum sensitiveness to steady current. There is

usually no difficulty, however, in obtaining sufficient sensitiveness of galvanometer to detect want of proportionality of the arms P Q R S of the bridge, so that a change of S by 1 in 200 will thus give a deflexion of convenient magnitude. It is, however, very advantageous to have a galvanometer whose period of oscillation can be varied at pleasure, so as to give about the same throw as steady deflexion for the chosen value of dS. One advantage which the method has is that there is no necessity to know the galvanometer or battery resistance. In the ordinary ballistic-galvanometer method it is very doubtful if the resistance of the galvanometer, unless it is measured at the time, is known within 1 to 2 per cent.

Further, the two essential parts of the determination θ and α may be made in quick succession, without any shifting of contacts or connexions and with the battery-current flowing continuously, leaving only the period of the galvanometer and its damping correction to be afterwards determined.

The method may be of service in the simultaneous determination of the resistance of, and joint capacity and inductance of, a submarine cable or telephone- or telegraph-line.

Discussion.

Prof. Perry asked what were the advantages of the method as compared with the Rayleigh-Sumpner method.

Mr. Blakesley thought that the correction for damping in the ballistic part of the experiment might be avoided, if, in the second part, the disturbance of balance due to the increment of S were measured by half the first throw of the needle on making the galvanometer circuit, instead of by the steady deflexion. He doubted whether reversing the current in the battery circuit would have just twice the effect of simply breaking the circuit.

In reply, Mr. Womack said he had not tried the method of reading suggested by Mr. Blakesley; but with regard to the reversing of the battery circuit, that was found to give in practice as nearly as possible twice the deflexion which resulted from simply breaking.

XXXI. *On the Determination of Thermal Conductivity and Emissivity.* *By* N. EUMORFOPOULOS, *B.Sc., Demonstrator in Physics, University College, London**.

IF a long metallic bar of uniform cross-section be heated at one end, and be left long enough to acquire a steady state, the distribution of temperature along it is usually expressed by

$$u_x = u_0 e^{-x\sqrt{\frac{hp}{ks}}},$$

where u_x is the excess of temperature over surroundings of a point on the bar at a distance x from the point whose excess of temperature is u_0, x being measured positively along the direction of decreasing temperatures;

h and k are the emissivity and conductivity respectively;

p is the periphery of the bar, and s the cross-section.

If the bar be of circular cross-section and r its radius, the above expression reduces to

$$u_x = u_0 e^{-x\sqrt{\frac{2h}{kr}}};$$

that is, if two bars have at one point of each the same temperature u_0, they will also have the same temperature u_x at distances x_1 and x_2 respectively, measured from these points, and such that

$$x_1\sqrt{\frac{h_1}{k_1 r_1}} = x_2\sqrt{\frac{h_2}{k_2 r_2}}.$$

If the bars are of different radii, but of the same material, and we accordingly write $h_1 = h_2$ and $k_1 = k_2$, we get

$$\frac{x_1}{x_2} = \sqrt{\frac{r_1}{r_2}}.$$

Some time ago Prof. G. C. Foster suggested to me, as a laboratory exercise, to undertake the verification of this relation.

The method adopted was to heat the ends of two such rods

* Read January 11, 1895.

in steam until they had acquired the steady state, and then by means of two thermoelectric joints (one on each rod) to find a series of isothermal points.

With two *brass* rods of radii $r_1=3{\cdot}4$ and $r_2=2{\cdot}6$ mm. respectively, the distances at which equal temperatures were found were as follows :—

$$\sqrt{\frac{r_2}{r_1}}=0{\cdot}87.$$

x_1.	x_2	$\frac{x_2}{x_1}$
20 mm.	13 mm.	·65
40	29	·72
60	42	·70
85	64	·75
110	78	·71
		Mean = ·71

Similar experiments with *copper* rods gave :—

$r_1=3{\cdot}35$ mm., $r_2=2{\cdot}45$ mm. $\sqrt{\frac{r_2}{r_1}}=0{\cdot}86.$

x_1.	x_2.	$\frac{x_2}{x_1}$.
20 mm.	18 mm.	·90
40	25	·62
60	34	·57
80	47	·59
100	61	·61

In the first value of $\frac{x_2}{x_1}$ some error has, no doubt, crept in.

Mean of last four = ·60.

Another set of experiments with the same rods gave :—

x_1.	x_2.	$\frac{x_2}{x}$.
20 mm.	10 mm.	·50
40	22	·55
20	11	·55
40	22	·55
20	12	·60
		Mean = ·55

To ascertain whether the copper rods really consisted of the same metal, their specific electrical resistances were compared. The result was

$$\frac{\rho_1}{\rho_2} = \cdot 80.$$

Assuming that the ratio of thermal conductivities was the same as that of electrical conductivities, this gives

$$\frac{x_2}{x_1} = \sqrt{\frac{k_2 r_2}{k_1 r_1}} = \cdot 77$$

as the calculated ratio of the distances at which equal temperatures should be found, instead of ·60 and ·55, the mean ratios actually found in the two sets of experiments already mentioned.

In order to test the trustworthiness of the method employed, one of the above rods was heated in the middle under the same conditions as before, and isothermal points found on each side of the heated portion. The results were satisfactory, as the following numbers show :—

x.	x_2.
20 mm.	19 mm.
40	40
60	61
80	81

At this point the experiments were abandoned for a time. They were subsequently resumed, when a Bunsen burner was used as the source of heat, in order to have a greater range of temperature. In the experiments mentioned before, and also in these, no screens were employed, a sheltered part of the laboratory being used. The following are some of the results obtained :—

German Silver Rods.

$$r_1 = 3 \cdot 0 \text{ mm.}, \quad r_2 = 2 \cdot 5 \text{ mm.}, \quad \sqrt{\frac{r_2}{r_1}} = \cdot 91.$$

Making $x_2 = 10 \cdot 3$ cm., $x_1 = 11 \cdot 95$ cm. as a mean of five determinations, or

$$\frac{x_2}{x_1} = \cdot 86.$$

As a mean of a large number of experiments, making $x_2 = 10{\cdot}0$ cm., x_1 was found to be 11·68 cm., or

$$\frac{x_2}{x_1} = {\cdot}86.$$

The arrangements were then slightly changed, so as to experiment on a hotter part of the rod, with the following result as a mean of a series of experiments :—

$x_2 = 10{\cdot}0$ cm., $x_1 = 11{\cdot}86$ cm. ; or

$$\frac{x_2}{x_1} = {\cdot}84.$$

The ratio of specific electrical resistances for these rods was found to be ·95 ; assuming this as the ratio of k_2 to k_1, we get

$$\frac{x_2}{x_1} = \sqrt{\frac{k_2 r_2}{k_1 r_1}} = 0{\cdot}89,$$

the mean result found by experiment being 0·85.

It is to be noted that there is not so much difference between the radii of the two rods as there was in the other cases.

Brass Rods, being the rods previously experimented on.
$x_1 = 65$ mm. ;
$x_2 = 47{\cdot}3$ mm. (mean of three determinations).

$$\frac{x_2}{x_1} = {\cdot}73, \quad \sqrt{\frac{r_2}{r_1}} = {\cdot}87.$$

Copper Rods, previously used.
$x_1 = 60$ mm. ;
$x_2 = 33$ mm. (mean of three determinations).

$$\frac{x_2}{x_1} = {\cdot}55, \quad \sqrt{\frac{r_2}{r_1}} = {\cdot}86.$$

Brass Rods. These are two new rods, which were used in experiments to be described later.

$$r_1 = 3{\cdot}21 \text{ mm.}, \quad r_2 = 1{\cdot}69 \text{ mm.}, \quad \sqrt{\frac{r_2}{r_1}} = {\cdot}73.$$

As a mean result of a series of experiments, for $x_2 = 5$ cm., x_1 was found to be 7·96 cm. ; or

$$\frac{x_2}{x_1} = {\cdot}63$$

It must be pointed out, however, that with the Bunsen burner the parts of the rod nearest the flame become visibly oxidized, this being especially noticeable in the case of the thicker rod—so much so that the increased emissivity so obtained can apparently overcome the greater transmission of heat of the thicker rod. Thus, with the above brass rods, 5 cm. on the thinner rod were found equivalent in this region to 5·2 cm. on the thicker; and after the heating had continued for some time 3·7 cm. on the thicker rod balanced 5 cm. on the thinner one. The ratio given above, ·63, is for parts of the rods that were not oxidized, and did not become so after several days' heating.

The general results obtained so far are :—

Rods.	r_1.	r_2.	$\sqrt{\frac{r_2}{r_1}}$.	$\frac{x_2}{x_1}$.
Brass	3·4	2·6	·87	·72
Brass	3·2	1·7	·73	·63
Copper	3·35	2·45	·86	·57
German silver .	3·0	2·5	·91	·85

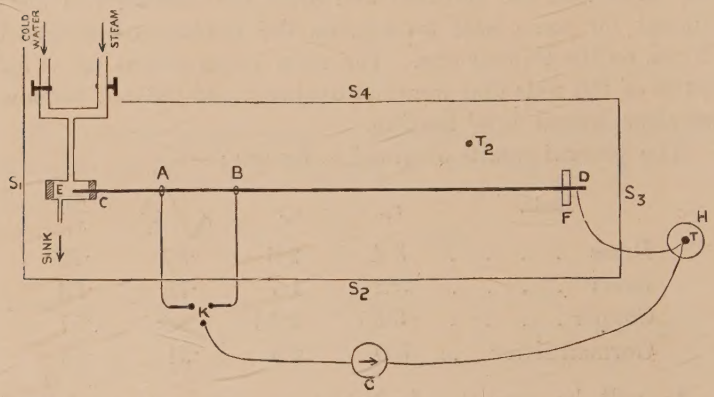
It will be seen that in every case the rate of fall of temperature along the thinner of the various pairs of rods compared is more rapid than would be inferred according to the usual formula from the distribution of temperature observed along the thicker rod. Consequently this formula cannot be legitimately used for the comparison of conductivities, unless the radii of the rods compared are equal and their surfaces in the same condition.

The only way of escaping from this conclusion is by supposing that the thermal conductivity of the smaller of each pair of rods was less than that of the thicker one. Although it seemed very unlikely that this should be so in the case of every pair, it could not be regarded as impossible. To settle the matter, it was decided to determine the absolute conductivity of at least one pair of rods.

For this purpose Ångström's method was adopted, in which one end of the rod is alternately heated and cooled, and the alternations of temperature observed at two points along the rod. Fig. 1 is a diagram of the apparatus used. C D is the rod, passing into a glass tube E, through which

cold water or steam can be passed as desired. E is held up by a clamp, and F is a clamp supporting the rod at the other end, no other clamps being used for the rod. S_1, S_2, S_3, S_4 are brown-paper screens, about 2 ft. high, resting on the table and open at the top; S_3 is about $1\frac{1}{2}$ ft. wide. A and B are the two points whose temperatures have to be measured.

Fig. 1.

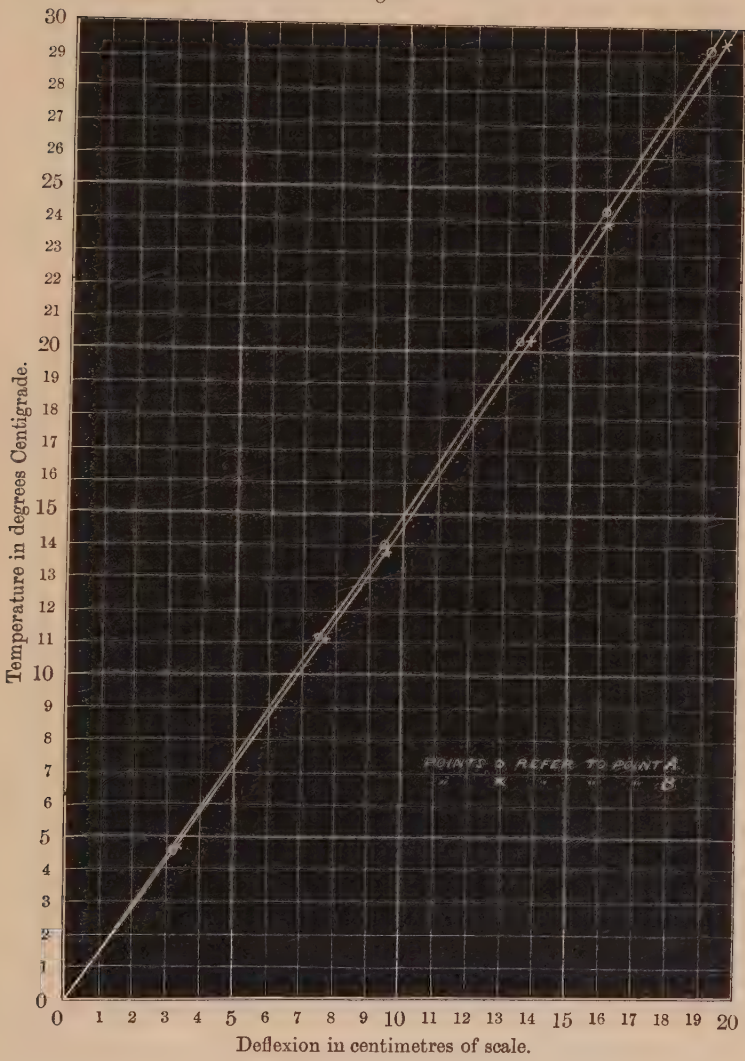


This was done by fine iron wires passing over the rod at the required point, and held down by suitable weights. The wires pass to a key K, through the galvanometer G, to a vessel H containing cold water. H contains the other junction, and a thermometer T. A brass wire connects D to H. T_2 is a thermometer reading the temperature of the air. Both thermometers were divided into fifths and read by a telescope.

As it was not possible to keep the temperature of the air constant, allowance had to be made for the change. As the temperature required is the excess of temperature of the rod over the air, this was done by merely subtracting the temperature of the air at the given moment from that of the rod at the same moment.

Experiment showed that it was possible to move the iron wire on the rod, and replace it, and obtain the same reading as previously; thus showing that the resistance of the joint did not appreciably alter. As the galvanometer was made somewhat sensitive, its sensitiveness was tested during the experiments by a Daniell cell passing through a large resist-

Fig. 2.



ance (about 13,000 ohms), the E.M.F. of the Daniell being compared with that of a Clark cell. Zero readings were also taken between each deflexion. The readings of the galvanometer were afterwards standardized, a known difference of

temperature being produced between the two junctions, and the deflexions observed. The following table gives an example of a standardization :—

Point A.		Point B.	
Diff. of Temp.	Galv. Defl.	Diff. of Temp.	Galv. Defl.
4·56	3·10	4·62	3·18
11·12	7·45	11·19	7·60
13·99	9·36	13·89	9·43
20·29	13·37	20·34	13·67
24·50	16·08	23·99	16·06
29·32	18·99	29·54	19·48

See fig. 2, which is plotted for these values.

When the temperature of the rod has settled down to its regular cyclic state, the excess of temperature u of any particular point at any instant t may be expressed by

$$u = a + b_1 \sin\left(2\pi \frac{t}{\mathrm{T}} + \beta_1\right) + b_2 \sin\left(4\pi \frac{t}{\mathrm{T}} + \beta_2\right) + b_3 \sin\left(6\pi \frac{t}{\mathrm{T}} + \beta\right) + \ldots,$$

where T is the time-period of the alternations.

The constants a, b_1, β_1, &c. are determined by the method of least squares from the observations made.

If a, b_1, β_1, &c. and a', b_1', β_1', &c. are the constants for the two points A and B respectively, and x the distance between them, then

$$b = b' e^{gx} \quad \text{and} \quad \beta - \beta' = qx$$

(see *Ann. Chim. Phys.* lxvii., 1863).

Hence g and q can be determined, and

$$gq = \frac{n\pi}{\frac{k}{c\delta}\mathrm{T}} \quad \text{and} \quad h = (g^2 - q^2)\frac{kr}{2},$$

where $n = 1, 2, 3$, &c., according as you use the constants of the 1st, 2nd, 3rd, &c. sine term ;

c = specific heat of the rod ;

δ = density of the rod.

Hence k and h can be determined from each sine term. In the experiments as carried out only the first, and perhaps the third, sine terms are available for these determinations, as the others are very small, the even terms being especially so. Besides these, there is the "a" term of the above formula (the constant term of the Fourier series), which is large, and represents the mean excess of temperature of the rod. From this term we get

$$\frac{a}{a'} = e^{x\sqrt{\frac{2h}{kr}}}.$$

If we use the previously determined value of k, we can get from this equation a second value of h. This will be referred to later on.

The rods experimented upon were three brass rods (commercial specimens) of roughly $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{3}{8}$ in. diameter; their lengths were about 3, 5, and 6 ft. respectively. They were cleaned, but not polished. The time-period of the alternations, and the positions of A and B, were chosen so that the cycle of changes gone through should be as nearly as possible the same in the three cases.

The specific heat was determined by heating a portion of each rod to 100° in a Regnault's apparatus in the usual way. The following are the results obtained :—

Diam. of Rod	$\frac{1}{8}$ in.	$\frac{1}{4}$ in.	$\frac{3}{8}$ in.
	·0937	·0943	·0945
	·0948	·0952	·0938
	·0953		·0942
Approx. rise of temp. of calorimeter . .	0·7	1·2	1·8 degr. Cent.

The mean result ·0945 was assumed for all the rods. No correction was made for change of specific heat with temperature.

The following readings are given to show how far they agree for different cycles on the same day :—

Point A.				Point B.			
I. Time: $1^h\ 0^m$.	II. $3^h\ 0^m$.	III. $4^h\ 0^m$.	Mean.	I. Time: $1^h\ 30^m$.	II. $3^h\ 30^m$.	III. $4^h\ 30^m$.	Mean.
20·15	19·85	19·85	19·95	8·83	8·86	8·85	8·85
17·16	16·93	16·91	17·00	8·88	9·00	8·99	8·96
13·87	13·69	13·74	13·77	8·75	8·78	8·82	8·78
11·25	11·05	10·97	11·09	8·26	8·31	8·27	8·28
.....	.....	.....	.....	.....	.....	.....	.....
3·16	3·05	3·06	3·09	3·86	3·94	3·88	3·89
2·70	2·59	2·56	2·62	3·44	3·45	3·47	3·45
2·29	2·23	2·16	2·23	3·03	3·13	3·09	3·08
1·97	1·88	1·83	1·89	2·66	2·78	2·75	2·73

The above readings are deflexions in centimetres : 1 cm. of scale corresponds to a rotation of the magnet through 11′, or 1 cm. corresponds to a difference of temperature between the two junctions of $1\frac{1}{2}$ degrees Cent.

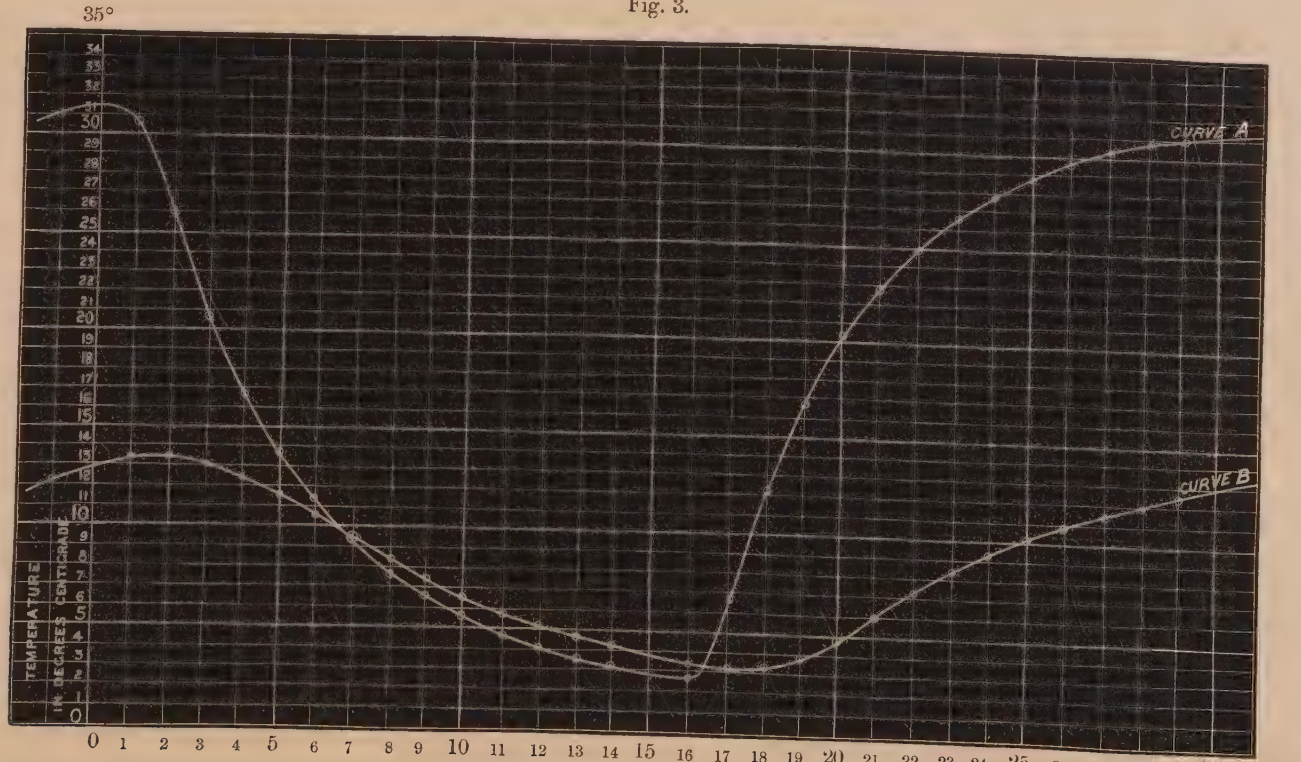
The following expressions show the series required to fit the temperatures obtained; they are the expressions for Set I. of Rod II., but the constants are of approximately the same magnitude with all the rods:—

$$\text{Temp. A} = 13{\cdot}95 + 12{\cdot}67 \sin\left(2\pi\frac{t}{T} + 2{\cdot}1412\right) + {\cdot}42 \sin\left(4\pi\frac{t}{T} + 3{\cdot}91\right)$$
$$+ 2{\cdot}18 \sin\left(6\pi\frac{t}{T} + 1{\cdot}13\right) + {\cdot}18 \sin\left(8\pi\frac{t}{T} + 3{\cdot}33\right)$$
$$+ {\cdot}80 \sin\left(10\pi\frac{t}{T} + {\cdot}56\right) + \ldots + {\cdot}31 \sin\left(14\pi\frac{t}{T} + {\cdot}28\right).$$

$$\text{Temp. B} = 6{\cdot}16 + 3{\cdot}92 \sin\left(2\pi\frac{t}{T} + 1{\cdot}3806\right) + {\cdot}10 \sin\left(4\pi\frac{t}{T} + 2{\cdot}41\right)$$
$$+ {\cdot}38 \sin\left(6\pi\frac{t}{T} - {\cdot}40\right) + {\cdot}04 \sin\left(8\pi\frac{t}{T} + 1{\cdot}14\right)$$
$$+ {\cdot}07 \sin\left(10\pi\frac{t}{T} - 1{\cdot}70\right) + \ldots + {\cdot}03 \sin\left(14\pi\frac{t}{T} - 2{\cdot}68\right).$$

As an example of the curve of temperatures obtained, see fig. 3. At the abscissa marked 0 min. cold water was turned

Fig. 3.


35°
TEMPERATURE IN DEGREES CENTIGRADE
CURVE A
CURVE B
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 minutes.
Time in minutes.

on, and at 15 min. steam. The points marked are experimental points. There are no points marked at 0 and 15 min. as I had to be in another part of the room at those instants. It will be noticed that the cold water or the steam (as the case may be) has done nearly all it can do, before the other is turned on, especially in the case of A. This can also be seen from the numbers given below.

The following are the details for each rod:—

Rod I. ($\frac{1}{8}$ inch).—Diam. = ·3377 cm. Density = 8·49. Specific electrical resistance = $7{\cdot}24 \times 10^{-6}$ ohms. 7 cms. between junctions. Time-period 10 min. Determination of temperature every half-minute.

	Set I.* (Mean of 3 cycles.) Excess Temperature in degrees.		Set II. (Mean of 3 cycles.) Excess Temperature in degrees.		Set III. (Mean of 4 cycles.) Excess Temperature in degrees.	
Time in mins.	A.	B.	A.	B.	A.	B.
0	—	—	—	—	—	—
½	22·25	9·13	22·27	9·26	19·82	8·67
1	15·91	8·88	15·95	8·98	14·05	8·35
1½	10·95	7·81	11·01	7·88	9·79	7·24
2	7·90	6·58	7·87	6·50	6·72	6·00
2½	5·67	5·29	5·68	5·29	4·83	4·71
3	4·35	4·35	4·27	4·29	3·42	3·79
3½	3·32	3·52	3·30	3·49	2·61	3·06
4	2·64	2·98	2·55	2·88	1·97	2·52
4½	2·05	2·36	2·02	2·31	1·56	2·01
5	—	—	—	—	—	—
5½	1·80	1·78	1·43	1·60	1·10	1·48
6	9·11	1·65	8·63	1·56	7·46	1·42
6½	15·47	2·58	14·95	2·52	13·97	2·36
7	19·16	4·06	18·95	3·97	17·59	3·81
7½	20·91	5·42	20·77	5·40	19·42	5·10
8	22·21	6·53	22·31	6·58	20·75	6·16
8½	23·08	7·41	23·31	7·42	21·65	6·99
9	23·69	7·95	23·96	8·16	22·49	7·65
9½	24·14	8·47	24·34	8·62	22·53	8·05
Mean Temp.	27°	19½°	27°	19½°	27½°	21°
Mean Temp. Air	14°		14°		16°	

* At the end of this set the joint B was found displaced through about 2 mm. away from A. This introduces a possible error of about 2½ p. c. in the distance, or 5 p. c. in the conductivity, while leaving the emissivity practically unaffected.

The results, in C.G.S. units, calculated from these numbers are :—

Conductivity (from 1st sine term).	Set I. [·2407]	Set II. ·2507	Set III. ·2541
Emissivity:—			
From "a" term.	·000321	·000331	·000319
„ 1st sine term.	·000397	·000403	·000405

Set II. is probably the best of the three.

Rod II. ($\frac{1}{4}$ inch).—Diam.=·6426 cm. Density=8·47. Specific electrical resistance = $7{\cdot}56 \times 10^{-6}$ ohms. 10 cms. between junctions. Time-period 20 min. Determination of temperature every minute.

	Set I. (Mean of 4 cycles.) Excess Temperature in degrees.		Set II. (Mean of 3 cycles.) Excess Temperature in degrees.	
Time in mins.	A.	B.	A.	B.
0	—	—	—	—
1	24·12	10·29	23·67	10·13
2	17·60	10·07	17·33	10·02
3	12·66	9·15	12·62	9·09
4	9·23	7·80	9·34	7·91
5	6·88	6·54	6·95	6·69
6	5·23	5·40	5·46	5·60
7	4·01	4·52	4·27	4·68
8	3·18	3·76	3·33	3 91
9	2·43	3·13	2·60	3·30
10	—	—	—	—
11	2·46	2·17	2·54	2·34
12	9·40	2·02	9·27	2·26
13	15·23	2·95	14·87	3·07
14	19·04	4·33	18·68	4·39
15	21·48	5·70	21·13	5·78
16	23·26	6·94	22·78	6·88
17	24·49	7·88	23·89	7·90
18	25·26	8·66	24·64	8·70
19	25·76	9·34	25·16	9·36
Mean Temp.....	26½°	18½°	28°	20½°
Mean Temp. Air.	12½°		14½°	

Whence by calculation :—

Conductivity (from 1st sine term).	Set I. ·2351	Set II. ·2338

Emissivity:—	Set I.	Set II.
From "a" term.	·000253	·000236
„ 1st sine term.	·000301	·000286

Set II. is the better of the two.

Rod III. ($\frac{3}{8}$ inch).—Diam. = ·9654 cm. Density = 8·49. Specific electrical resistance = $7{\cdot}71 \times 10^{-6}$ ohms. 12 cms. between junctions. Time-period 30 min. Determination of temperature every minute.

Time in mins.	Set I. (Mean of 3 cycles.) Excess Temperature in degrees. A.	Set I. B.	Set II. (Mean of 2 cycles.) Excess Temperature in degrees. A.	Set II. B.	Set III. (Mean of 3 cycles.) Excess Temperature in degrees. A.	Set III. B.
0	—	—	—	—	—	—
1	30·85	13·15	30·83	13·21	30·15	13·02
2	25·96	13·32	26·20	13·38	25·59	13·18
3	20·86	13·05	21·15	13·14	20·57	12·92
4	16·75	12·35	17·09	12·45	16·54	12·19
5	13·52	11·36	13·91	11·47	13·41	11·19
6	11·19	10·31	11·51	10·41	11·04	10·15
7	9·28	9·27	9·55	9·34	9·11	9·15
8	7·80	8·27	7·96	8·34	7·62	8·15
9	6·57	7·32	6·74	7·45	6·37	7·26
10	5·63	6·51	5·66	6·63	5·35	6·51
11	4·80	5·77	4·81	5·85	4·50	5·72
12	4·10	5·17	4·11	5·17	3·81	5·06
13	3·60	4·64	3·55	4·60	3·25	4·49
14	3·11	4·12	3·07	4·12	2·75	3·97
15	—	—	—	—	—	—
16	2·65	3·34	2·44	3·28	2·16	3·19
17	6·67	3·07	6·59	3·03	6·29	2·92
18	12·19	3·21	12·28	3·15	11·91	3·03
19	16·75	3·87	16·85	3·81	16·44	3·67
20	20·23	4·93	20·25	4·88	19·79	4·71
21	22·83	6·10	22·81	6·03	22·40	5·85
22	24·87	7·24	24·85	7·17	24·39	7·08
23	26·45	8·28	26·43	8·25	25·97	8·07
24	27·70	9·21	27·56	9·17	27·13	9·08
25	28·68	10·03	28·44	10·04	27·90	9·89
26	29·55	10·82	29·22	10·76	28·69	10·62
27	30·10	11·37	30·04	11·30	29·25	11·24
28	30·56	11·91	30·31	11·84	29·80	11·75
29	30·97	12·37	30·77	12·35	30·17	12·19
Mean Temp.	30°	21°	31°	22°	31½°	23°
Mean Temp. Air.	13°		14°		15°	

From which by calculation we get:—

	Set I.	Set II.	Set III.
Conductivity (from 1st sine term).	·2374	·2386	·2399
Emissivity:—			
From "a" term.	·000208	·000208	·000205
„ 1st sine term.	·000251	·000237	·000237

Set I. is the best of the three.

The third sine term is, in the case of B, too small to give reliable results, but they have been calculated in some cases, and curiously enough the conductivity always comes out lower than that given by the first term. The following have been calculated :—

	Rod I.		Rod II.	Rod III.		
	Set II.	Set III.	Set I.	Set I.	Set II.	Set III.
Conductivity—1st sine term.	·251	·254	·235	·237	·239	·240
3rd „ „	·229	·242	·233	·222	·229	·227
Emissivity — 1st sine term.	·00040	·00040	·00030	·00025	·00024	·00024
3rd „ „	·00038	·00025	·00027	·00024	·00026	·00031

Taking the mean of the chief results obtained, we get the following table :—

	Rod I.	Rod II.	Rod III.
Conductivity . . .	·2524	·2344	·2386
Emissivity:—			
From "a" term.	·000324	·000244	·000207
„ 1st sine term.	·000402	·000293	·000242

It hence appears that the emissivity of a body depends to a large extent on the form and dimensions of that body, and that the formula for conduction of heat along a rod, as usually given in text-books, can only be used in a very restricted sense.

It will be noticed that the two emissivities (deduced from the "a" term and the sine term) are to one another approximately

as 1 to 1·2. The ratio appears to be too uniform to be due to accidental error of experiment, but the meaning of the two values is not quite clear. In order to obtain further information, the $\frac{3}{8}$ inch rod was heated at one end by steam under approximately the same conditions as before, and the E.M.F. of a german-silver-iron couple at different points on the rod determined by the compensation method in the ordinary way. A preliminary set was taken, which gave about the same results as the following more accurate set (readings were taken going down the rod and then going up):—

Position on rod in cm.	Bridge-Reading in cm.		
	Down.	Up.	Mean.
0	73·8	73·7	73·75
5	53·1	52·6	52·85
10	38·0	38·0	38·0
15	27·6	27·5	27·55
20	19·8	20·0	19·9
25	14·4	14·7	14·55
30	10·6	10·8	10·7
35	7·8	7·8	7·8

Temperature of air about $10\frac{1}{2}°$.

1 cm. of bridge-wire corresponds to rather less than one degree.

Dividing the above into two lots, and taking $k = ·2386$, we get (this method is sufficiently accurate for this purpose):—

Mean temp.	h.
54°	·000247
42°	·000240
33°	·000231
27°	·000229

The mean temperature of the rod in the experiments by Ångström's method was about 27°, and the two emissivities found were ·000207 and ·000242, so that the steady-state emissivity comes just about halfway between the two. The rod, however, had become somewhat tarnished during the summer, and had to be cleaned again, which may or may not have made some difference. Further experiments will be needed to settle the point conclusively.

When the last part of the experiments was in progress, Mr. A. W. Porter, B.Sc., drew my attention to Messrs. Ayrton and Kilgour's paper before the Royal Society on the variation of emissivity with the diameter of the wires experimented upon. The results obtained corroborate one another, and also corroborate results that had been obtained some time ago by Péclet, and which are recorded in his *Traité de la Chaleur*. Péclet appears to have carried out a large number of experiments on emissivity, the best account of which is found in the 3rd edition (not the 4th) in vol. i. and a long note at the end of vol. iii. (published in 1860–61).

In conclusion I must express my obligations to Prof. Foster and Mr. Porter for much help given in the course of these experiments.

XXXII. *On the Influence of the Dimensions of a Body on the Thermal Emission from its Surface.* *By* Alfred W. Porter, *B.Sc., Demonstrator of Physics, University College, London**.

In discussing the rate at which heat passes outwards from a conducting body into the medium in which it is immersed, it is usually assumed that (for small excesses of temperature of the body above its surroundings) it may be taken as proportional to the excess of temperature. This law is followed, as far as is known, if the body is in a vacuum and loses heat simply by radiation between its own and surrounding surfaces. If, however, it is immersed in a medium, such as air, which carries away heat by conduction and convection as well as by radiation, the problem becomes more complicated, and to assume that the whole effect may be treated as radiation only does not give results which are even an approximation to those obtained experimentally. For example, on this assumption the amount passing outwards from unit area of the surface per second per unit excess of temperature (*i. e.* the "emissivity")

* Read January 11, 1895.

should be independent of the size of the body. Results obtained by Péclet from experiments on cylinders and spheres of different sizes show that this constant depends materially upon the sizes of the bodies experimented upon. Péclet's formulæ connecting the rate of emission (exclusive of the radiation effect) with the radius (r) for 0° C. excess are :—

For a horizontal infinitely long cylinder of brass . . . . $2{\cdot}058+\frac{{\cdot}0382}{r}$

For a sphere . . . . . $1{\cdot}774+\frac{{\cdot}13}{r}$

in which kilogrammes, metres, hours are the units employed. Other formulæ are given by him which it is unnecessary to quote here.

These results do not seem to have attracted much notice, as they are not given in any of the text-books of Physics. They do, however, appear in a book compiled for practical men by Box *, from which the formula for the sphere is quoted, on Professor Ayrton's authority, in Everett's 'Illustrations of the C.G.S. System of Units' (1891 edition, p. 133) †.

* 'A Practical Treatise on Heat for the use of Engineers and Architects,' by Thomas Box. (London, E. and F. N. Spon, 2nd edition, 1876.) This book does not appear to be generally known to physicists; and, judging from recent references to it, I gather that still less is it realized that the author's data are to a great extent obtained from Péclet, although he states the source to which he is indebted in his preface.

† A mistake occurs in Everett as well as in a paper on the same subject published later by Professor Ayrton and Mr. Kilgour in the Phil. Trans. for 1892, in which Everett's statement is quoted. The formulæ are given in Box as

$$\cdot421+\frac{\cdot307}{r} \text{ for the cylinder}$$

and

$$\cdot3634+\frac{1\cdot0476}{r} \text{ for the sphere,}$$

in which the units are the pound, foot, and hour, *the radius being however in inches.*

Translated into C.G.S. units, they become

$$\text{Cylinder}: \left[\cdot572+\frac{1\cdot061}{r}\right]\times10^{-4},$$

$$\text{Sphere}: \quad \cdot00004928+\frac{\cdot0003609}{r}.$$

Experiments on thin wires by Messrs. Ayrton and Kilgour have confirmed the fact that the emissivity can be expressed empirically through a considerable range of radius in the form given by Péclet; and experiments on rods which have been in progress in this Laboratory since 1891, an account of which was read by Mr. Eumorfopoulos before the Physical Society on the same day as this paper, lead to the same conclusion.

It was these that first called my attention to the subject; and in order to account approximately for them and the results elsewhere obtained, I propose here to examine the results of supposing the loss to only in part follow the law of radiation, the remainder being assumed to follow the law of conduction.

The rate of loss due to radiation will be proportional to the excess of the temperature of the body above that of its enclosure, and if we reckon temperatures from that of the enclosure, we may write the rate due to this cause

$$h\,\theta_a,$$

where h is the radiation-constant and θ_a is the temperature at the surface of the body.

The remainder, which is to include both true conductive and also convective loss, will be proportional to the temperature-gradient in the medium close to the surface of the body. It may on this assumption be represented by

$$-c\left.\frac{d\theta}{dr}\right|_a,$$

where $\left.\frac{d\theta}{dr}\right|_a$ is the temperature-gradient in the medium at the bounding surface of the body, and c is a positive constant which will be referred to as the convection-conductivity.

In Everett, and in Ayrton and Kilgour's paper, the latter appears as

$$\cdot 0004928 + \frac{\cdot 0003609}{r}.$$

Further, it is not clear from Everett whether the formula he gives refers to air-effect *plus* radiation or to one of these alone. The specification of "blackened sphere" would lead one to suppose that either the total effect or else the radiation only is meant, since the air-effect has been shown to be independent of the *nature* of the surface. On reference to Péclet, the formula is seen to represent the *air-effect alone*.

The total rate of loss is hence

$$h\theta_a - c\left.\frac{d\theta}{dr}\right|_a,$$

which may be written

$$\left[h - c\frac{\left.\frac{d\theta}{dr}\right|_a}{\theta_a}\right]\theta_a.$$

The expression inside the brackets is the quantity called the "emissivity" in the usual treatment of the problem. On the above assumption, far from being a constant, it is seen to be a thing whose value will vary with every modification of the experiment by which it is sought to be determined.

To fix ideas, take the case of a cylindrical rod or wire of radius "a" heated uniformly at all points and maintained at constant temperature by mechanism the nature of which is of no consequence. Let it be surrounded by a coaxal cylindrical sheath of radius R maintained at a constant temperature, which will be taken as the zero of temperature, the intervening space being filled with conducting or pseudo-conducting material. The differential equation to be satisfied by the temperature in the medium is

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} = 0,$$

since with the above conditions everything is symmetrical with regard to, and uniform parallel to, the axis of the cylinder; and the solution of this which satisfies the boundary conditions is

$$\theta = \theta_a \frac{\log \mathrm{R} - \log r}{\log \mathrm{R} - \log a};$$

whence the emissivity (e) becomes

$$h + \frac{c}{a(\log \mathrm{R} - \log a)},$$

or for convenience of calculation,

$$h + \frac{b}{a(\log_{10} \mathrm{R} - \log_{10} a)}.$$

This, then, is presumably an approximation to a theoretic

formula for the case of experiments like those of Messrs. Ayrton and Kilgour referred to before; and the accompanying Table I. and curves (fig. 1) (pp. 348 & 349) show how closely, by a suitable choice of constants, it can represent the experimental values. The constants I determined by the method of least squares; this I considered it advisable to do, because the experimental values are not sufficiently precise to enable one to draw a smooth curve through them with anything like certainty. In particular I may mention that their own empirical formulæ fail to fit in with the experimental values yielded by the wire of radius ·0037; and these values are similarly shunned by the formulæ which I give.

The first term in each of my formulæ should represent the radiation-constant. It must be noted, however, that for such very thin wires radiation forms only a small portion of the whole emission; and therefore, without having results of greater accuracy from which to deduce the formulæ, it is not possible to assign the value of this term with definiteness. All the values for it are, however, higher than those obtained by observers who have experimented on the emission from surfaces *in vacuo*, as the following data show:—

Radiation Values.

Quoted from Everett.	Bottomley: Sooted globe in Sprengel vacuum	·000125	for excess of	83°·6.
		·000095	„ „	32°·7.
	Bottomley: Silvered and highly polished globe	·0000354	„ „	72°·1.
		·0000296	„ „	43°·7.
	Nichol (published by Tait): Pressure of air = 10 mm. of mercury	·000057	„ „	42°.

As regards the other constant "b" ($=\cdot4343\,c$), we have

At 300° . .	$b=\cdot0696\times10^{-3}$,	$c=\cdot16\times10^{-3}$,
200° . .	$b=\cdot0620\times10^{-3}$,	$c=\cdot143\times10^{-3}$,
150° . .	$b=\cdot0605\times10^{-3}$,	$c=\cdot139\times10^{-3}$,
100° . .	$b=\cdot0549\times10^{-3}$,	$c=\cdot126\times10^{-3}$.

Péclet's results were derived from experiments on five cylinders: only two of them are available for my purpose, since the others were either of different material or different surface. They were short cylinders of unpolished brass, terminated by hemispherical ends, allowance for which was

TABLE I.—Thermal Emissivities of Wires at different Temperatures obtained experimentally by Messrs. Ayrton and Kilgour, compared with calculated Values.

a. Radius of wire, in centimetres.	Temperature 300°. Experimental values $\times 10^3$.	Temperature 300°. Values calculated from Ayrton and Kilgour's formula.	Temperature 300°. Values calculated from Formula given in this paper.	Temperature 200°. Experimental values $\times 10^3$.	Temperature 200°. Values calculated from Ayrton and Kilgour's formula.	Temperature 200°. Values calculated from Formula given in this paper.	Temperature 150°. Experimental values $\times 10^3$.	Temperature 150°. Values calculated from formula given in this paper.	Temperature 100°. Experimental values $\times 10^3$.	Temperature 100°. Values calculated from Ayrton and Kilgour's formula.	Temperature 100°. Values calculated from Formula given in this paper.
·00155	14·40	14·54	14·69	12·78	13·03	12·80	11·87	12·30	10·85	11·04	11·22
·00255	9·85	9·18	9·39	9·07	8·26	8·47	8·60	8·07	7·90	7·07	7·38
·00370	5·84	6·68	6·92	5·38	6·04	6·27	5·09	5·92	4·55	5·20	5·43
·0076	3·84	3·82	3·92	3·46	3·49	3·59	3·21	3·31	2·93	3·05	3·06
·0103	3·35	3·12	3·12	3·08	2·88	2·88	2·94	2·61	2·80	2·53	2·43
·0118	2·84	2·86	2·83	2·59	2·65	2·61	2·45	2·36	2·30	2·33	2·19
·0141	2·61	2·58	2·48	2·36	2·40	2·31	2·22	2·06	2·05	2·12	1·93
·0178	2·29	2·28	2·11	2·14	2·13	1·96	2·03	1·73	1·89	1·90	1·63
	Formulæ referred to above. Ayrton and Kilgour's:— $e = \cdot 001135 + \frac{\cdot 016084}{D}$, where D is diameter in mils. Formula from which the last column is calculated:— $e = \left[\cdot 29 + \frac{\cdot 0696}{a \log\left(\frac{2\cdot 54}{a}\right)}\right] \times 10^{-3}$.			Formulæ referred to above. Ayrton and Kilgour's:— $e = \cdot 001111 + \frac{\cdot 014303}{D}$, where D is diameter in mils. Formula from which the last column is calculated:— $e = \left[\cdot 36 + \frac{\cdot 0620}{a \log\left(\frac{2\cdot 54}{a}\right)}\right] \times 10^{-3}$.			No formula given by Ayrton and Kilgour. Formula from which the second column is calculated:— $e = \left[\cdot 16 + \frac{\cdot 0605}{a \log\left(\frac{2\cdot 54}{a}\right)}\right] \times 10^{-3}$.		Formulæ referred to above. Ayrton and Kilgour's:— $e = \cdot 001036 + \frac{\cdot 012078}{D}$, where D is diameter in mils. Formula from which the last column is calculated:— $e = \left[\cdot 20 + \frac{\cdot 0549}{a \log\left(\frac{2\cdot 54}{a}\right)}\right] \times 10^{-3}$.	

Fig. 1.

Thermal Emissivities of Wires at Different Temperatures.

Ordinates represent Emissivities.
Abscissæ represent Radii of Wires.

Messrs. Ayrton and Kilgour's Experimental Values indicated thus:

Points on their Empirical Curves indicated thus:

Curves represented by formulæ given in this paper indicated thus:

made from the experiments on spheres. Péclet, therefore, considered his results to apply to infinitely long cylinders. Their radii were 5·1 and 6·9 cm. respectively, and measurements were made on them in a cylindrical enclosure 40 cm. radius and 100 cm. long. His results are :—

Radius.	Experimental results.
5·1	·0000867
6·9	·0000803

and the formula which I derive from these is

$$e = \cdot 0000383 + \frac{\cdot 00022}{a(\log_{10} 40 - \log_{10} a)}.$$

In large cylinders like these radiation plays a more conspicuous part, and the value of the first term in my formula which represents it is in very good accordance with values obtained by direct methods. The value of "b" is, however, very different from that obtained from Ayrton and Kilgour's experiments; and the conclusion is forced upon one that although to consider "b" a constant may serve as long as the enclosure remains the same, it will by no means suffice to take it the same constant under such widely different conditions as obtain in these two sets of experiments. The character of the convective flow is evidently totally different in the two cases.

The same conclusion is arrived at by an examination of results obtained from experiments on spheres. The formula for a sphere, found by means of the same assumption as before, is

$$e = h + \frac{c\mathrm{R}}{a(\mathrm{R} - a)}.$$

Péclet experimented on three spheres in the same enclosure; and assuming the enclosure to behave like a spherical one of 45 centim. radius, the formula becomes

$$e = \cdot 00003845 + \frac{\cdot 000417 \times 45}{a(45 - a)}.$$

The experimental and calculated values are here given:—

Radius in centim.	Experimental value of e for 0° excess.	Calculated value from above formula.	Calculated by Péclet's own formula for air-effect and adding radiation.
7·465	·0001051	·0001054	·0001048
10·58	·0000898	899	906
15·38	·0000801	796	800

It will be observed that the radiation-constant is practically the same as that derived from his cylinder results*, while the value of c is rather less.

Experiments on spheres (both of 2 centim. radius) have also been made by MacFarlane† and extended by Bottomley‡, both using the same enclosure, which was of "large dimensions" (*Bottomley*).

Extrapolation from Péclet's results for a sphere of 2 centim. radius in his enclosure gives :—

Using his formula.	Using formula given above.
·000237.	·000256

MacFarlane's results for a blackened sphere (Péclet's were dull) give ·000238 for 0° excess. Bottomley's results give about ·000260 for 0° excess, showing that the formula derived from Péclet's results is roughly applicable here.

Bottomley also experimented on the same sphere in a spherical enclosure of 5 centim. radius. If the formulæ hitherto given in this paper were applicable to this case, the

* Péclet endeavoured to estimate the radiation independently by means of a thermopile, and deducted it from the total emission to obtain the air-effect. The value he obtained was only ·0000072 for 0° excess, a value so much smaller than received values that I have ignored it entirely, and dealt only with the total effect.

† D. MacFarlane, Proc. Roy. Soc. 1892.

‡ J. T. Bottomley "On Thermal Radiation in Absolute Measure," Trans. Roy. Soc. 1892.

emissivity ought to increase with decrease of radius of enclosure. Bottomley's results show a marked *decrease*. The value at 17° for 0° excess is not more than ·00011 after correcting approximately for temperature.

Since the true radiation was determined separately it is possible to find the value of "*c*" for these experiments, although only one sphere was employed. I subjoin a table:—

From	*c*.
Péclet's results from spheres	·000417.
MacFarlane's and Bottomley's results in *large* enclosure, assuming Bottomley's value for radiation ...	about ·0004.
Bottomley's results for sooted sphere in small enclosure	·000192 for excess of 85° C. 181 „ „ 65° C. 168 „ „ 45° C. 160 „ „ 35° C.
Bottomley's results for silvered and highly polished sphere in small enclosure	·000168 for excess of 65° C.

Place against these the values obtained from cylinders:—

From	*c*.
Péclet's results in large enclosure ...	·000508 for 0° excess.
Ayrton and Kilgour's, in enclosure of 2·54 centim. radius	·000160 with wire at 300° C. 143 „ „ 200° C. 139 „ „ 150° C. 126 „ „ 100° C.

In order to throw light on this question, I have started experiments, in conjunction with Mr. Eumorfopoulos, on cylindrical rods in cylindrical enclosures of different radii. Experiments made so far are as follows:—

A brass rod ·483 centim. radius has soldered on it *two* thermoelectric junctions of iron and german-silver at a distance apart of 10 centim., each of which is part of a couple whose other junction is kept cold in a water-pot. The rod is heated at one end by steam until the steady state of temperature is attained, and the ratio of the temperatures (reckoning from enclosure temperature as zero) of the two junctions is measured successively with different water-jackets embracing the rod.

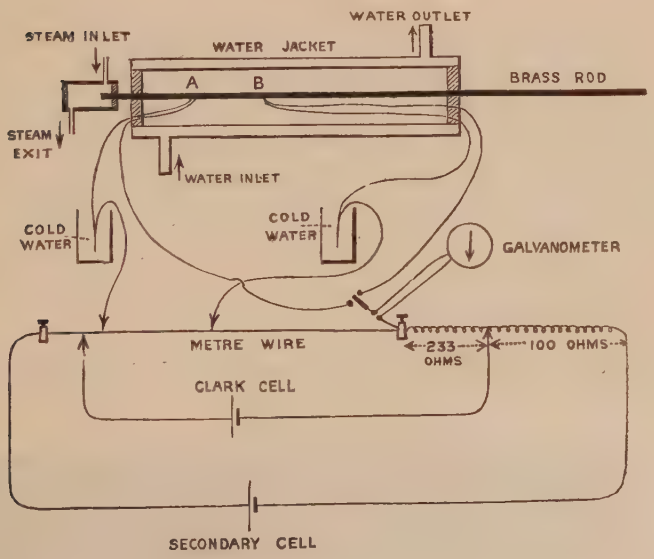
The general arrangement is shown in fig. 2 (p. 353).

Tap-water is passed through the water-jacket at such a rate that its temperature as it enters is sensibly the same as when it leaves the jacket. This temperature is read by a thermometer, as also are the temperatures of the cold junctions. Then either the junction at A and its corresponding cold junction, or the junction at B and its corresponding cold junction, can be connected to a potentiometer, and the thermoelectric E.M.F. in each case determined.

The potentiometer was standardized from time to time by a Clark's cell which is so connected (switches not shown) that the same galvanometer does for all three operations.

The ratio of the thermoelectric E.M.F.'s is approximately the same as the ratio of the excess temperatures of the points

Fig. 2.



A and B after it has been corrected for slight differences of temperature between the two water-baths and the water-jacket. Two jackets have been employed so far, and compared with each other and with the bar unjacketed. In this last case the bar is merely shielded from the ruder form of draught by brown-paper screens.

The results are given in the subjoined table:—

R. Inner radius of enclosure in centim.	Excess temperature of A. θ_A.	Excess temperature of B. θ_B.	Average excess.	Emissivity at stated mean excess temperature.	Emissivity approximately corrected to one temperature.
Indefinitely great.	54·0	30·5	42·2	·0001882	·000191
5·15	57·0	33·6	45·3	·0001610	·000162
1·6	61·1	36·6	48·8	·0001514	·000151

The values of emissivity here given have been calculated from the formula

$$e = \frac{kr}{2}\left[\frac{\log \theta_A - \log \theta_B}{\cdot 4343 \times x}\right],$$

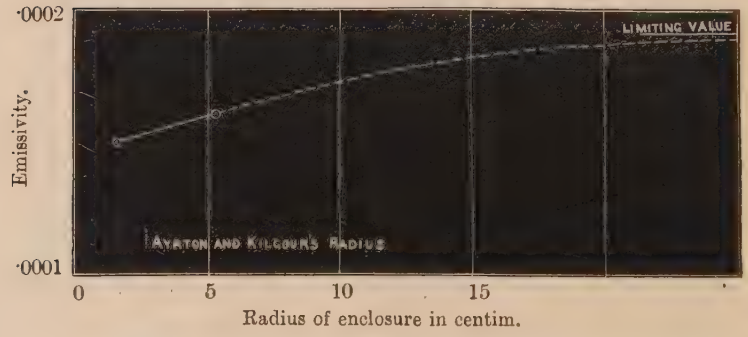
in which $x=10$ centim., and the value of k taken is that determined by Mr. Eumorfopoulos for the same rod by Ångström's method, viz. ·2386. The emissivity is therefore in each case an average value for the range of temperature between the two points. The actual temperatures were ascertained by standardizing the thermoelectric couples in water-baths of known temperatures.

The above results are also shown plotted in fig. 3. An

Fig. 3.

Variation of Emissivity of Cylindrical Brass Rod with the Radius of its Cylindrical Enclosure.

Radius of rod = ·483 centim.



elastic curve (bent steel lath) has been made to pass through the two results for definite dimensions and extended provisionally by a dotted line so as to rise asymptotically to the highest value. It is unsafe to rest too much upon these few results: more will need to be obtained before the law of variation can be stated with certainty: the following paragraph must be considered therefore as merely provisional. Take the two definite values and also (by *inter*polation) the value for an enclosure of 2·54 centim. radius (that employed by Messrs. Ayrton and Kilgour) and find the values of "*b*." They are:—

Radius of enclosure. R.	Radius of rod. *a*.	*b*.
1·6	·483	$\cdot029\times10^{-3}$
2·54	·483	·0418 „
5·15	·483	·0643 „

The value derived from A. and K.'s results at 60° is $\cdot047\times10^{-3}$. As their thin wires blocked up the enclosure less than the thick rod here employed, it is natural that the value obtained from them should be greater than that given above.

The formula which I have given for a cylinder appears to hold through a far wider range than that given by Messrs. Ayrton and Kilgour. Take for example their formula for a wire at 100°. The emissivity for it can never (for any radius) fall lower than

·00107.

Calculating from my formula we get

·00036

for a rod of ·483 centim. radius at 100°. On reduction to 60° it becomes about

·00025,

which is much more within sight of the value ·000154 which is obtained from fig. 3. Considering the violent nature of the extrapolation here made, the agreement is probably as close as could reasonably be expected.

Conclusions.

i. That the theoretic assumption made in this paper gives results which are accordant through a wide range of radius of rod with experimental results obtained under similar conditions as regards enclosure: and in this respect is far superior to the usual assumption which gives no account whatever of variation of the value of emissivity with radius.

ii. That if the freedom permitted to convection effects be varied, as it will be if the enclosure be changed, it is necessary to consider the convecto-conduction constant as varying with the changed conditions according to a law which can only be found by a complete series of experiments made with enclosures of different dimensions.

iii. I conclude that the enclosing boundary is as important a factor in determining the value of the emissivity as the size of the body itself: and that therefore in any collection of data (such as Everett's) it is very necessary to specify the exact nature of the enclosure in which the experiments were conducted; and, further, that all determinations that have been made of this constant and published with imperfect description of all the boundaries are of little scientific value.

Finally, I must express my thanks to Prof. G. C. Foster for kindly advice and suggestions given from time to time in connexion with this matter.

Discussion.

Prof. Carey Foster thought that in demonstrating the influence of the enclosure Mr. Porter had established an important point.

Prof. Ayrton agreed as to the importance of the influence of the enclosure. Some years ago he had noticed that when the diameter of a wire was small, the power required to keep it red-hot by the passage of a current was nearly independent of the diameter. This would also follow from Péclet's formula. He urged that in such experiments as those of Mr. Porter and Mr. Eumorfopoulos, the conductivity and

emissivity, which were functions of the temperature, should not be assumed constant along the bar.

Mr. TROTTER objected to the use of the term emissivity as including loss of heat by contact with the air, in addition to the loss by radiation.

Mr. GRIFFITHS said that in some of his experiments, where a wire conveying an electric current was immersed in a liquid in order to heat the latter, the rise of the temperature of the wire above that of the liquid was found to be nearly independent of the diameter of the wire.

Mr. EUMORFOPOULOS, replying to Prof. Ayrton's criticism, said that in each case his comparison had been between portions of bars in which the range of temperature was the same. Moreover, the variation of the emissivity and conductivity with temperature, as found by other observers, would be quite insufficient to account for his results.

Mr. PORTER said that the term emissivity had come to be accepted as referring to all heat lost at the surface of a body, whether by radiation or by conduction and convection. In that sense he had used the term.

Prof. RÜCKER thought that emissivity, in this sense, was not a good term, but to change now would probably only make greater confusion.

XXXIII. *On the Passage of an Oscillator Wave-train through a Plate of Conducting Dielectric.* By GEORGE UDNY YULE, *Demonstrator in Applied Mathematics, University College, London.*

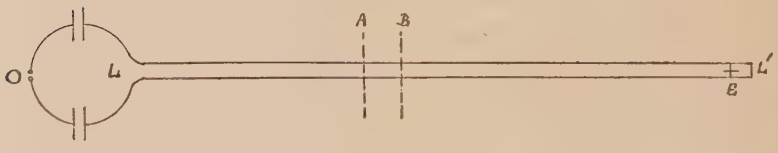
I. Introduction.
II. The Phenomena at the First Surface of the Plate.
III. The Phenomena at the Second Surface of the Plate.
IV. The Intensity of the Transmitted Ray.
V. The Intensity of the Reflected Ray.
VI. Discussion of an Experimental Case and a Correction.
VII. The Numerical Value of some of the previous Results.
Synopsis of Symbols.

I. *Introduction.*

THE problem treated in these pages was suggested by the following experimental results, for a fuller account of which I may refer to a previous paper †.

A series of wave-trains generated by an oscillator O were allowed to propagate themselves along a double-wire circuit LL′ about 100 metres in length. At the middle of the circuit the wires were run through a jar containing a certain depth AB of distilled water, alcohol, or a very dilute electrolyte.

Fig. 1.



The intensity of the ray transmitted by this slightly absorbent layer was determined by an electrometer E placed at a quarter wave-length (2·25 metres) from the closed end of the circuit. This intensity varied periodically with the thickness of the

* Read January 11, 1895.

† Phil. Mag. vol. xxxvi. p. 531, Dec. 1893. Prelim. Paper in the Proc. Roy. Soc. vol. liv. p. 96. A sketch of an approximate theory was given in the Phil. Mag. paper, but several assumptions there made were unjustifiable.

layer, as in the analogous case of a ray of light and a "thin plate."

The layer AB was what Boltzmann has termed a "conducting dielectric," *i. e.* both its conducting and dielectric properties were of importance. The theory of a non-conducting plate is much simpler: a problem equivalent to it has recently been treated by Dr. E. H. Barton *, to whose work I shall have several occasions to refer. In the more complex problem I have assumed, we have simply a plane-fronted damped wave-train travelling through an insulating medium and falling at normal incidence on an infinite slab of conducting dielectric. The magnetic permeability of both the plate and the surrounding medium are taken as unity.

The theoretical transmission-curve obtained on these assumptions does not agree well with the experimental one, the divergence being the same in sign and order of magnitude as that noticed by Dr. Barton in his case. We have in fact idealized the experimental facts too far in endeavouring to simplify the analysis: the electrolyte was not at all infinite in extent, and at least one important correction is obviously necessary (section VI.). The experiments can only be regarded as a very rough illustration of the problem, and as giving the *raison d'être* of this paper.

The general results obtained seem to be of considerable interest. The intensities of the reflected rays, the phase-changes, and so on, for damped wave-trains reflected from such a plate differ from those for steady rays in some cases very considerably.

For convenience and brevity, the surrounding medium is hereinafter generally referred to as "the dielectric," and the conducting slab as "the plate" or "electrolyte."

II. *The Phenomena at the First Surface of the Plate.*

In this section we will deal with the phenomena occurring at the first surface of the "electrolyte" only.

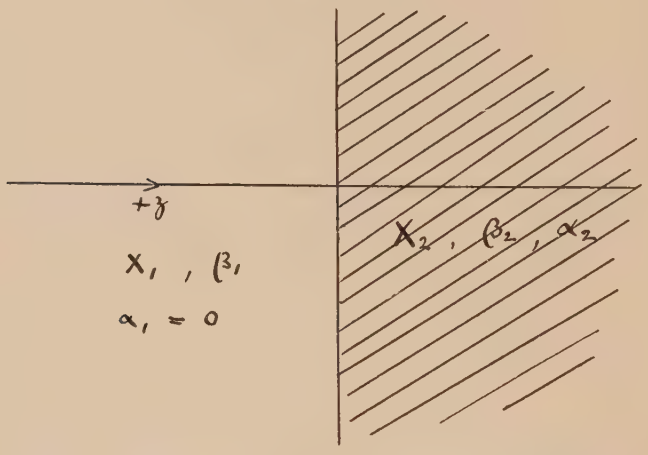
The direction of propagation of the ray is chosen as the

* Preliminary Paper, Proc. Roy. Soc. vol. liv. p. 85; more fully in Thesis for the D.Sc. London, 1894, or Wied. *Ann.* vol. liii. p. 513, 1894. Final paper, Proc. Roy. Soc., read April 1893 (not yet published).

positive direction of the axis of z and the origin is taken indefinitely close to the interface.

The suffix 1 always refers to the dielectric and the suffix 2 to the plate.

Fig. 2.



Let X_1 be the electric force in the dielectric and X_2 that in the plate. Under the limitations we have set ourselves—plane waves and non-conducting dielectric—Maxwell's equations assume the simple forms

$$\beta_1 \frac{d^2X_1}{dt^2} = \frac{d^2X_1}{dz^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\left(\alpha_2 + \beta_2 \frac{d}{dt}\right)\frac{dX_2}{dt} = \frac{d^2X_2}{dz^2}, \quad . \quad . \quad . \quad . \quad (2)$$

where we have abbreviated Maxwell's notation by putting

$$\beta_1 = \mu_1\kappa_1, \quad \beta_2 = \mu_2\kappa_2, \quad \alpha_2 = 4\pi\mu_2 C_2, \quad . \quad . \quad (3)$$

μ_2 being the magnetic permeability of the plate, κ_2 its dielectric constant, C_2 its conductivity; μ_1 κ_1 are the same constants referring to the dielectric, C_1 being zero. As already stated, we will only take the case of both media being non-magnetic, or

$$\mu_2 = \mu_1 = 1.$$

As a solution of equations (1) and (2) we select

$$\left.\begin{aligned} X_1 &= A_1 e^{p(t - \sqrt{\beta_1} z)} + A_1' e^{p(t + \sqrt{\beta_1} z)} \\ X_2 &= B_2 e^{p(t - qz)} \end{aligned}\right\} \quad . \quad . \quad (4)$$

where t must be not less than zero. p is either wholly imaginary or complex according as the incident ray is steady or damped: q is as yet undetermined; A_1 is the amplitude of the incident wave-train, A_1' of the reflected wave-train, and B_2 of the transmitted wave-train.

Inserting the value of X_2 given in (4) in equation (2), and carrying out the differentiation, we get

$$q = \sqrt{\beta_2}\left(1+\frac{\alpha_2}{\beta_2 p}\right)^{\frac{1}{2}}, \quad \ldots\ldots \quad (5)$$

so q is also partly imaginary.

To determine $\frac{A_1'}{A_1}$ and $\frac{B_2}{A_1}$ we require another pair of equations. These will be given us by the interface conditions: the conditions namely, that there must be no discontinuous jump in the values of either the tangential electric force or the tangential magnetic force, at any time, in passing from the dielectric to the electrolyte. That is, we must have

$$\left.\begin{aligned} X_1 &= X_2, \\ \frac{dX_1}{dz} &= \frac{dX_2}{dz}, \end{aligned}\right\} \quad \ldots\ldots \quad (6)$$

where

$$z=0, \quad t=\text{anything}.$$

Inserting the values of X_1 and X_2 in (6) we get, putting $z=0$,

$$\left.\begin{aligned} A_1+A_1' &= B_2, \\ p\sqrt{\beta_1}\,(-A_1+A_1') &= -pqB_2; \end{aligned}\right\} \quad \ldots \quad (7)$$

and solving these two equations for $\frac{A_1'}{A_1}$, $\frac{B_2}{A_1}$,

$$\frac{A_1'}{A_1} = -\frac{\frac{q}{\sqrt{\beta_1}}-1}{\frac{q}{\sqrt{\beta_1}}+1},$$

$$\frac{B_2}{A_1} = \frac{2\sqrt{\beta_1}}{\sqrt{\beta_1}+q}.$$

If we write for brevity

$$\left.\begin{aligned} \lambda &= \frac{\sqrt{\beta_1}}{q}, \\ \frac{A_1'}{A_1} &= -\frac{1-\lambda}{1+\lambda}, \\ \frac{B_2}{A_1} &= \frac{2\lambda}{1+\lambda}, \end{aligned}\right\} \quad \ldots\ldots \quad (8)$$

λ contains q, which is partly imaginary; so we must separate the real and imaginary quantities and rationalize. Let p_1 be the real part of p, and p_2 the imaginary part, so that

$$p = -p_1 + p_2 i,$$

and let us write

$$\left.\begin{aligned} p_1 &= \eta \cos \chi, \\ p_2 &= \eta \sin \chi. \end{aligned}\right\} \quad \ldots\ldots \quad (9)$$

In this notation then

$$\left.\begin{aligned} p &= -\eta e^{-i\chi}, \\ \text{period} &= \frac{2\pi}{p_2}, \\ \text{logarithmic decrement} &= \frac{2\pi p_1}{p_2}; \end{aligned}\right\}$$

and using the value of q in (5),

$$\lambda = \sqrt{\frac{\beta_1}{\beta_2}} \frac{1}{\left(1 - \frac{\alpha_2}{\eta\beta_2} e^{i\chi}\right)^{\frac{1}{2}}}.$$

The denominator of this expression is

$$\left(1 - \frac{\alpha_2}{\eta\beta_2} \cos\chi - i \frac{\alpha_2}{\eta\beta_2} \sin\chi\right)^{\frac{1}{2}},$$

so if we put

$$\left.\begin{aligned} \rho \cos\theta &= 1 - \frac{\alpha_2}{\eta\beta_2} \cos\chi \\ \rho \sin\theta &= \frac{\alpha_2}{\eta\beta_2} \sin\chi \end{aligned}\right\}, \quad \ldots \quad (10)$$

or

$$\rho = \left[1 - \frac{\alpha_2}{p_2\beta_2} \sin 2\chi + \left(\frac{\alpha_2}{p_2\beta_2} \sin\chi\right)^2\right]^{\frac{1}{2}}$$

$$\tan\theta = \frac{\frac{\alpha_2}{p_2\beta_2} \sin^2\chi}{1 - \frac{1}{2}\frac{\alpha_2}{p_2\beta_2} \sin 2\chi},$$

the denominator becomes $\rho e^{-\frac{i\theta}{2}}$. Writing

$$r=\sqrt{\frac{\beta_1}{\beta_2}}\frac{1}{\sqrt{\rho}}, \quad \ldots\ldots \quad (11)$$

as a further abbreviation we have

$$\lambda=re^{\frac{i\theta}{2}}.$$

We can now proceed to rationalize the values of $\frac{A_1'}{A_1}$ and $\frac{B_2}{A_1}$ given in (8). Inserting our value of λ,

$$\frac{A_1'}{A_1}=-\frac{1-re^{\frac{i\theta}{2}}}{1+re^{\frac{i\theta}{2}}}=-\frac{1-r\cos\frac{\theta}{2}-ir\sin\frac{\theta}{2}}{1+r\cos\frac{\theta}{2}+ir\sin\frac{\theta}{2}}.$$

Multiplying numerator and denominator by

$$1+r\cos\frac{\theta}{2}-ir\sin\frac{\theta}{2},$$

this becomes

$$-\frac{1-r^2-2ir\sin\frac{\theta}{2}}{1+r^2+2r\cos\frac{\theta}{2}},$$

and multiplying again by $1-r^2+2ir\sin\frac{\theta}{2}$,

$$\frac{A_1'}{A_1}=-\frac{1+r^4-2r^2\cos\theta}{\left(1+r^2+2r\cos\frac{\theta}{2}\right)\left(1-r^2+2ir\sin\frac{\theta}{2}\right)}.$$

If then

$$\tan\psi=\frac{2r\sin\frac{\theta}{2}}{1-r^2}, \quad \ldots\ldots \quad (12)$$

we have finally

$$\frac{A_1'}{A_1}=-\frac{(1+r^4-2r^2\cos\theta)^{\frac{1}{2}}}{1+r^2+2r\cos\frac{\theta}{2}}e^{-i\psi};$$

that is, the ray reflected from the first surface of the plate undergoes a phase-change of $(-\psi)$, the reduction factor or ratio of the *amplitude* of the reflected to that of the incident ray being

$$b=-\frac{(1+r^4-2r^2\cos\theta)^{\frac{1}{2}}}{1+r^2+2r\cos\frac{\theta}{2}}. \quad \ldots\ldots \quad (13)$$

b is a function of the period and damping of the wave-train as well as of the properties of the reflector. If the wave-train be not damped, we have

$$\left.\begin{aligned} \chi &= 90^\circ, \\ \rho &= \sqrt{1+\left(\frac{\alpha_2}{p_2\beta_2}\right)^2}, \\ \cos\theta &= \frac{1}{\rho}, \end{aligned}\right\} \quad . \quad . \quad . \quad (14)$$

so the expression for b remains unaltered in form, b being still a function of the period.

But if the conductivity of the electrolyte or α_2 be zero, the angle θ becomes likewise zero, ρ is unity, and we get the ordinary value for b, namely,

$$b = \frac{1-\sqrt{\frac{\beta_1}{\beta_2}}}{1+\sqrt{\frac{\beta_1}{\beta_2}}}, \quad . \quad . \quad . \quad . \quad . \quad (15)$$

the period and damping of the incident ray having vanished from the expression.

The phase-change ψ is zero for either a perfect insulator or a perfect conductor; θ becoming indefinitely small in the first case, and r in the second.

The refracted wave is dealt with in exactly the same manner. Taking the value of $\frac{B_2}{A_1}$ from (8), and substituting in it the value of λ from (11) we have

$$\frac{B_2}{A_1} = \frac{2\lambda}{\lambda+1} = \frac{2re^{\frac{i\theta}{2}}}{1+re^{\frac{i\theta}{2}}}$$

$$= \frac{2re^{\frac{i\theta}{2}}\left(1+r\cos\frac{\theta}{2}-ri\sin\frac{\theta}{2}\right)}{1+r^2+2r\cos\frac{\theta}{2}}.$$

If we write

$$\tan\psi' = \frac{r\sin\frac{\theta}{2}}{1+r\cos\frac{\theta}{2}}, \quad . \quad . \quad . \quad . \quad (16)$$

then $$\frac{B_2}{A_1} = \frac{2r}{\sqrt{1+r^2+2r\cos\dfrac{\theta}{2}}} e^{i(\theta/2-\psi')}.$$

Thus for the transmitted ray at the dielectric-electrolyte surface, the change of phase is $\frac{\theta}{2}-\psi'$, and the ratio of its amplitude to that of the incident ray or c is

$$c = \frac{2r}{\sqrt{1+r^2+2r\cos\dfrac{\theta}{2}}}. \quad . \quad . \quad . \quad . \quad (17)$$

c, like b, is a function of the period and damping of the incident ray ; like b it remains a function of the period even for an undamped ray ; and like b it becomes a function of the dielectric constants only, if the " electrolyte " be a perfect insulator, the expression in (17) becoming

$$c = \frac{2\sqrt{\dfrac{\beta_1}{\beta_2}}}{1+\sqrt{\dfrac{\beta_1}{\beta_2}}}. \quad . \quad . \quad . \quad . \quad . \quad (17a)$$

We have now sufficient data to determine the speed of propagation and wave-length in the electrolyte, before going on to deal with the reflexions and refractions at the second surface. Referring back to equation (4) for X_2, and substituting the values we have determined for B_2, p, and q, we have

$$X_2 = cA_1 e^{i(\theta/2-\psi')} e^{(-p_1+ip_2)t} e^{\eta\sqrt{\beta_2\rho}\{\cos(\chi+\theta/2)-i\sin(\chi+\theta/2)\}z},$$

or retaining only the real terms

$$X_2 = cA_1 e^{-p_1 t+\eta\sqrt{\beta_2\rho}\cos\left(\chi+\frac{\theta}{2}\right)z} \cos\left\{p_2 t-\eta\sqrt{\beta_2\rho}\sin\left(\chi+\frac{\theta}{2}\right)z + \frac{\theta}{2}-\psi'\right\}. \quad . \quad . \quad . \quad (18)$$

Hence the speed of the wave in the electrolyte is

$$V_2 = \frac{p_2}{\eta\sqrt{\beta_2\rho}\sin\left(\chi+\dfrac{\theta}{2}\right)} = \frac{\sin\chi}{\sqrt{\beta_2\rho}\sin\left(\chi+\dfrac{\theta}{2}\right)}, \quad . \quad (19)$$

and the wave-length

$$\lambda_2 = \frac{2\pi}{\eta \sqrt{\beta_2 \rho} \sin\left(\chi + \frac{\theta}{2}\right)} \quad . \quad . \quad . \quad . \quad (20)$$

But for the incident wave-train

$$\lambda_1 = \frac{2\pi}{p_2 \sqrt{\beta_1}},$$

therefore

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{\beta_1}{\beta_2}} \frac{p_2}{\eta \sqrt{\rho} \sin\left(\chi + \frac{\theta}{2}\right)}$$

$$= \sqrt{\frac{\beta_1}{\beta_2}} \frac{\sin \chi}{\sqrt{\rho} \sin\left(\chi + \frac{\theta}{2}\right)} \quad . \quad . \quad . \quad . \quad (21)$$

ρ and θ being both functions of α_2 and of the period, and χ being a function of the rate of damping of the wave-train, $\frac{\lambda_2}{\lambda_1}$ is (like b and c and the phase-changes) a function of period, damping, and conductivity, except when the latter is zero: when we have simply

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{\beta_1}{\beta_2}} \quad . \quad . \quad . \quad . \quad . \quad (21a)$$

III. *The Phenomena at the Second Surface of the Plate.*

We now proceed to the inverse case where the wave is passing from the electrolyte into the dielectric.

If X_2 represent as before the electric force in the electrolyte, and X_3 represent the electric force in the second dielectric, we may now write

$$X_2 = B_1 e^{p(t-qz)} + B_1' e^{p(t+qz)}, \quad . \quad . \quad . \quad . \quad (22)$$

$$X_3 = A_3 e^{p(t-\sqrt{\beta_1} z)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

the wave in the electrolyte consisting of a direct and a reflected train, and that in the dielectric of a direct train only.

Applying the interface conditions as in the last section, we get for $z=0$

$$B_1 + B_1' = A_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$\frac{dX_2}{dz} = \frac{dX_3}{dz} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Carrying out the differentiation of (25) and putting $z=0$,

$$-p\sqrt{\beta_1}A_3=pq(-B_1+B_1'),$$

or

$$B_1-B_1'=A_3\frac{\sqrt{\beta_1}}{q}=A_3\lambda. \quad . \quad . \quad . \quad (26)$$

Solving for $\frac{B_1'}{B_1}$ and $\frac{A_3}{B_1}$ from (24) and (26) we get

$$\left.\begin{aligned}\frac{A_3}{B_1}&=\frac{2}{1+\lambda}\\ \frac{B_1'}{B_1}&=\frac{1-\lambda}{1+\lambda}\end{aligned}\right\}. \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Using our previous notation of

$$\lambda=re^{\frac{i\theta}{2}}$$

and rationalizing, we obtain finally for the transmitted ray

$$\frac{A_3}{B_1}=\frac{2}{\sqrt{1+r^2+2r\cos\frac{\theta}{2}}}e^{-i\psi'},$$

so that the phase-change for the wave emerging from the electrolyte is $-\psi'$ (equation 16), and the reduction factor

$$f=\frac{2}{\sqrt{1+r^2+2r\cos\frac{\theta}{2}}}. \quad . \quad . \quad . \quad . \quad (28)$$

The expression for $\frac{B_1'}{B_1}$ in (27) is the same as that for $\frac{A_1'}{A_1}$ in (8) except for the reversal of sign, so the phase-change at the second surface of the electrolyte is the same $(-\psi)$ in magnitude and sign as at the first surface, and the reduction factor

$$e=-b. \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

If the electrolyte be replaced by an insulator, f takes the ordinary form

$$f=\frac{2}{1+\sqrt{\frac{\beta_1}{\beta_2}}}, \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and the relation

$$cf=1-b^2$$

holds. In the general case there is no such relation.

IV. *The Intensity of the Transmitted Ray.*

We have now all the data necessary for treating the general case of a wave-train passing through a plate of finite thickness. We will first take the incident wave-train and follow its history in detail to obtain the expressions for the rays emergent after 0, 2, 4, . . . &c. internal reflexions. Let the incident wave-train be given by

$$A\, e^{p(t-\sqrt{\beta_1}x)}$$

from the time $t=0$ onwards, p being a complex quantity as defined by (9). Passing through the first surface and reaching the second the wave becomes (d being the thickness of the electrolyte)

$$A\, c\, e^{p(t-qd)}\, e^{i(\theta/2-\psi')},$$

and emerges in the form

$$Acf\, e^{-pqd}\, e^{i(\theta/2-2\psi')}\, e^{p(t-\sqrt{\beta_1}x)}. \quad . \quad . \quad . \quad (I.)$$

If we take the instant at which the incident wave first strikes the plate as the origin of time, the wave-train (I.) begins to emerge at time

$$t_2 = \frac{d\,.\,\eta\,.\,\sqrt{\beta_2\rho}\,.\sin\left(\chi+\frac{\theta}{2}\right)}{p_2}, \quad . \quad . \quad . \quad (31)$$

by equation (19).

The wave reflected from the second surface at the time that (I.) emerges is

$$-A\,.\,bc\, e^{-pqd}\, e^{p(t+qx)}\, e^{i(\theta/2-\psi'-\psi)},$$

which becomes on reflexion at the first surface

$$A\,.\,b^2c\, e^{-2pqd}\, e^{p(t-qx)}\, e^{i(\theta/2-\psi'-\psi)},$$

and emerges from the second surface at time $3t_2$ as

$$Ab^2cf\, e^{-3pqd}\, e^{p(t-\sqrt{\beta_1}x)}\, e^{i(\theta-2\psi'-2\psi)}. \quad . \quad . \quad . \quad (II.)$$

This is the second emergent ray, or the ray emerging after two internal reflexions. The third emergent ray will be

$$Ab^4cf\, e^{-5pqd}\, e^{i(\theta/2-2\psi'-4\psi)}\, e^{p(t-\sqrt{\beta_1}x)}, \quad . \quad . \quad . \quad (III.)$$

emerging at time $5t_2$; and so on for the others. The $(n+1)$th emergent ray, or the ray emerging after $2n$ internal reflexions, is

$$Ab^{2n}cfe^{-(2n+1)pqd}e^{i(\theta/2-2\psi'-2n\psi)}e^{p(t-\sqrt{\beta_1}x)},$$

which emerges at time $(2n+1)t_2$. Remembering that

$$p=-p_1+p_2i=\eta(-\cos\chi+i\sin\chi),$$

$$q=\frac{\sqrt{\beta_1}}{\lambda}=\sqrt{\beta_2}\rho\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right),$$

this becomes

$$Acfb^{2n}e^{(2n+1)\kappa d}e^{-p_1t}e^{i[p_2t+\alpha-(2n+1)(p_2t_2+\psi)]}, \quad . \quad . \quad . \quad (32)$$

where we have written for brevity

$$\left.\begin{aligned}\eta\sqrt{\beta_2\rho}\cos\left(\frac{\theta}{2}+\chi\right)&=\kappa,\\ \frac{\theta}{2}-2\psi'+\psi&=\alpha.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Finally, if we take the initial form of the wave-train to be given by the sine terms, we may write this ray emerging after $2n$ internal reflexions, y_n, as

$$y_n=\mathrm{A}\,.\,cf.\,l^{2n}e^{(2n+1)\kappa d}e^{-p_1t}\sin\left[p_2t+\alpha-\overline{2n+1}(p_2t_2+\psi)\right]. \quad (34)$$

However small ψ may be it should evidently be retained, as it may become of importance owing to the multiple reflexions.

The wave-train y_n emerges at time $\overline{2n+1}t_2$. From $\overline{2n+1}t_2$ to $\overline{2n+3}t_2$, y_0 to y_n inclusive are the only wave-trains in existence. The total energy of any wave-train varies as the time-integral of the square of the "displacement"

$$(y), \text{ or } (y_0+y_1+y_2+\ldots+y_n).$$

We will call this time-integral the *intensity* of the wave-train (or set of wave-trains forming the ray), though we are not using intensity in its ordinary sense.

To get the intensity of the ray transmitted through the plate, we shall then have to sum up a series of integrals partly of squares $(y_0^2, y_1^2, \ldots y_n^2)$ and partly of products $(2y_m y_n$ &c.) :

that is to say, if we call the intensity of the transmitted ray I_t,

$$I_t = \sum_{n=0}^{n=\infty} \int_{\overline{2n+1}t_2}^{\infty} y_n^2\, dt + 2 \sum_{m=0}^{m=n-1} \sum_{n=1}^{n=\infty} \int_{\overline{2n+1}t_2}^{\infty} y_m\, y_n\, dt. \quad . \quad . \quad (35)$$

In the experimental case the deflexions of the electrometer at E, fig. 1, are proportional to I_t.

We have now to get out the integrals in (35) and sum them. We will first multiply out the product $y_m\, y_n$ from the expression for y_n given in (34). This can then be integrated, and the integral of y_n^2 obtained by putting $m=n$.

Writing, for brevity,

$$p_2 t_2 + \psi = \delta, \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$\begin{aligned} y_m\, y_n &= A^2 c^2 f^2\, b^{2(n+m)} e^{-2p_1 t} e^{2(m+n+1)\kappa d} \\ &\quad \times \sin\,(p_2 t + \alpha - \overline{2n+1}\delta)\, \sin\,(p_2 t + \alpha - \overline{2m+1}\delta) \\ &= \tfrac{1}{2} A^2 c^2 f^2 b^{2(n+m)} e^{-2p_1 t} e^{2(m+n+1)\kappa d} \\ &\quad \times \{\cos\,[2(m-n)\delta] - \cos\,2[p_2 t + \alpha - (m+n+1)\delta]\}. \quad (37) \end{aligned}$$

Integrating,

$$\int_{\overline{2n+1}t_2}^{\infty} y_m\, y_n\, .\, dt = \tfrac{1}{2} A^2 c^2 f^2 b^{2(m+n)} e^{2(m+n+1)\kappa d}$$

$$\times \left[\frac{-e^{-2p_1 t} \cos 2(m-n)\delta}{2p_1} + \frac{e^{-2p_1 t}}{2\eta} \cos\{2(p_2 t + \alpha) + \chi - 2(m+n+1)\delta\} \right]_{\overline{2n+1}t_2}^{\infty}$$

The expression in the square brackets vanishes at the upper limit; so we have, substituting $\eta \cos\chi$ for p_1 in the denominator of the first fraction,

$$\int_{\overline{2n+1}t_2}^{\infty} y_m\, y_n\, .\, dt = \frac{1}{4\eta} A^2 c^2 f^2 b^{2(m+n)} e^{2(m+n+1)\kappa d}$$

$$\times e^{-2(2n+1)p_1 t_2} \left\{ \frac{\cos 2(m-n)\delta}{\cos\chi} - \cos\,[2(n-m)p_2 t_2 + 2\alpha + \chi - 2(m+n+1)\psi] \right\}. \quad (38)$$

If we put $m=n$, we get

$$\int_{\overline{2n+1}t_2}^{\infty} y_n^2\, dt = \frac{1}{4\eta} A^2 c^2 f^2 b^{4n} e^{(4n+2)\kappa d} e^{-(4n+2)p_1 t_2}$$

$$\times \{\sec\chi - \cos\,[2\alpha + \chi - (4n+2)\psi]\}. \quad (39)$$

Having now got the integrals, we must next perform the summations indicated in (35). Let us take the y_n^2 integrals first: we have to sum the expression on the right-hand side of (39) from $n=0$ to $n=\infty$. The two terms in the bracket are best taken separately: the first forms a simple geometric series whose sum is

$$\frac{1}{4\eta}\mathrm{A}^2 . c^2 f^2 . \frac{e^{2(\kappa d - p_1 t_2)} \sec \chi}{1 - b^4 e^{4(\kappa d - p_1 t_2)}} . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

The second term requires different treatment, as the cosine is a function of n. We may take the cosine to be the real part of

$$e^{i(2\alpha + \chi - \overline{4n+2}\psi)},$$

and thus obtain again a geometrical series, the sum of which is

$$\frac{1}{4\eta}\mathrm{A}^2 . c^2 f^2 . \frac{e^{2(\kappa d - p_1 t_2)} . e^{i(2\alpha + \chi - 2\psi)}}{1 - b^4 e^{4(\kappa d - p_1 t_2)} e^{-4i\psi}} .$$

This is rationalized by multiplying numerator and denominator by $1 - b^4 e^{4(\kappa d - p_1 t_2)} e^{4i\psi}$; and to further abbreviate the expression we will write

$$\left.\begin{aligned} b e^{\kappa d - 2 p_1 t_2} &= \xi, \\ b e^{\kappa d} &= \omega, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

thus obtaining for the sum of the second terms of (39),

$$\frac{1}{4\eta}\mathrm{A}^2 c^2 f^2 . \frac{\omega \xi}{b^2} . e^{i(2\alpha + \chi - 2\psi)} \frac{1 - \omega^2 \xi^2 e^{4i\psi}}{1 - 2\omega^2 \xi^2 \cos 4\psi + \omega^4 \xi^4} .$$

Remembering that

$$\alpha = \frac{\theta}{2} - 2\psi' + \psi,$$

and writing

$$\theta - 4\psi' + \chi = \phi, \quad . \quad . \quad . \quad . \quad . \quad (42)$$

this becomes, retaining the real terms only,

$$\frac{1}{4\eta}\mathrm{A}^2 c^2 f^2 . \frac{\omega \xi}{b^2} . \frac{\cos \phi - \omega^2 \xi^2 \cos (\phi + 4\psi)}{1 - 2\omega^2 \xi^2 \cos 4\psi + \omega^4 \xi^4} . \quad . \quad (43)$$

Adding (40) and (42) together, we get for I_1, the portion of the transmitted intensity due to the square terms,

$$\mathrm{I}_1 = \mathrm{A}^2 \frac{c^2 f^2 . \omega \xi}{4\eta b^2} \left\{ \frac{\sec \chi}{1 - \omega^2 \xi^2} - \frac{\cos \phi - \omega^2 \xi^2 \cos (\phi + 4\psi)}{1 - 2\omega^2 \xi^2 \cos 4\psi + \omega^4 \xi^4} \right\} . \quad (44)$$

We have now to sum the product terms. We require

$$I_2 = 2 \sum_{n=1}^{n=\infty} \sum_{m=0}^{m=n-1} \int_{2n+1 t_2}^{\infty} y_m y_n \, dt$$
$$= I_2' - I_2'',$$

say; I_2' being the sum of the terms multiplied by $\sec\chi$, and I_2'' the sum of the second terms. The integrated expression is given in (38). Taking the $\sec\chi$ terms first,

$$I_2' = \frac{A^2 . c^2 f^2}{2\eta b^2} \sum_{n=1}^{n=\infty} \omega^{2n+2} e^{-(4n+2)p_1 t_2} \sec\chi \sum_{m=0}^{m=n-1} \omega^{2m} \cos 2(m-n)\delta ;$$

$\cos 2(m-n)\delta$ may be treated as the real part of $e^{2i(m-n)\delta}$, or

$$\omega^{2m} \cos 2(m-n)\delta = e^{-2in\delta}(\omega^2 e^{2i\delta})^m,$$

which is a geometric series whose sum from 0 to $n-1$ is

$$e^{-2in\delta} \frac{1-(\omega^2 e^{2i\delta})^n}{1-\omega^2 e^{2i\delta}}.$$

Multiplying numerator and denominator by $1-\omega^2 e^{-2i\delta}$ to rationalize, and retaining only the real quantities in the numerator, this becomes

$$\frac{\cos 2n\delta - \omega^{2n} - \omega^2 \cos(2n+2)\delta + \omega^{2n+2} \cos 2\delta}{1-2\omega^2 \cos 2\delta + \omega^4}.$$

Therefore

$$I_2' = \frac{A^2 . c^2 f^2}{2\eta b^2} \frac{\sec\chi}{1-2\omega^2 \cos 2\delta + \omega^4} \sum_{n=1}^{n=\infty} (\cos 2n\delta - \omega^{2n}$$
$$- \omega^2 \cos(2n+2)\delta + \omega^{2n+2} \cos 2\delta) \omega^{2n+2} e^{-(4n+2)p_1 t_2}.$$

The first and third terms in the bracket may be summed together, as also the second and fourth. Carrying out the summation as before, we get

$$I_2' = \frac{A^2 c^2 f^2}{2\eta b^2} \frac{\sec\chi . \omega\xi}{1-2\omega^2 \cos 2\delta + \omega^4} \left\{ \frac{\xi^2(\cos 2\delta - \xi^2) - \omega^2\xi^2(\cos 4\delta - \xi^2 \cos}{1-2\xi^2 \cos 2\delta + \xi^4} \right.$$
$$\left. - (1-\omega^2 \cos 2\delta) \frac{\omega^2\xi^2}{1-\omega^2} \right.$$

Bringing the two fractions to a common denominator, $(1-2\omega^2 \cos 2\delta + \omega^4)$ divides out, and

$$I_2' = \frac{A^2 c^2 f^2}{2\eta b^2} \frac{\sec\chi . \omega\xi}{1-\omega^2\xi^2} \frac{\xi^2(\cos 2\delta - \xi^2)}{1-2\xi^2 \cos 2\delta + \xi^4} \quad . \quad . \quad (45)$$

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1894-95.

February 9th, 1894.

Prof. A. W. Rücker, President, in the Chair.

A paper was read by Mr. Owen Glynne Jones, B.Sc., on "The Viscosity of Liquids."

February 23rd, 1894.

Prof. A. W. Rücker, President, in the Chair.

Mr. J. W. Kearton was elected a Member of the Society.

A note on "A New Electrical Theorem" was read by Mr. T. H. Blakesley.

Prof. C. V. Boys, F.R.S., read a note on "The Attachment of Quartz Fibres."

Mr. Littlewood read a note on "A Method of Determining Refractive Indices."

March 9th, 1894.

Prof. A. W. Rücker, President, in the Chair.

Prof. O. Henrici, F.R.S., made a communication on "Mathematical Calculating Machines, especially a New Harmonic Analyser."

Mr. H. Wilde, F.R.S., exhibited and described his "Magnetarium."

April 13th, 1894.

Prof. A. W. Rücker, President, in the Chair.

Mr. W. G. Rhodes was elected a Member of the Society.

A discussion upon Prof. Henrici's paper given at the last Meeting took place, and Mr. A. Sharp, B.Sc., read a paper on "An Harmonic Analyser."

Mr. P. L. Gray, B.Sc., read a paper on "The Minimum Temperature of Visibility."

April 27th, 1894.

Prof. A. W. Rücker, President, in the Chair.

Dr. C. V. Burton read a paper "On the Mechanism of Electrical Conduction."

A paper by Prof. S. P. Thompson, F.R.S., and Mr. Miles Walker, "On the Design and Winding of Alternate-Current Electromagnets," was read.

Major R. L. Hippesley, R.E., read a paper on "A Graphical Method of Constructing the Curves of Current in Electromagnets and Transformers."

May 11th, 1894.

Mr. Walter Baily, Vice-President, in the Chair.

A communication "On Electromagnetic Induction in Plane, Cylindrical, and Spherical Current Sheets, and its Representation by Moving Trails of Images" was made by G. H. Bryan, M.A.

A paper "On Dielectrics" was read by Mr. Rollo Appleyard.

Prof. Minchin made a communication "On the Behaviour of certain Bodies in presence of Electromagnetic Oscillations."

May 25th, 1894.

Prof. A. W. Rücker, President, in the Chair.

Prof. W. Ramsay, F.R.S., read a paper "On the Passage of Hydrogen through a Palladium Septum."

Prof. W. Ramsay, F.R.S., and Mr. E. C. C. Baly communicated a paper "On the Expansion of Rarefied Gases."

June 8th, 1894.

Prof. A. W. Rücker, President, in the Chair.

A discussion took place upon the paper by Prof. Ramsay and Mr. E. C. C. Baly, read at the last Meeting.

A paper "On the Isothermals of Ether," by Mr. Rose Innes, was read.

June 22nd, 1894.

Prof. W. E. Ayrton, Past President, in the Chair.

Captain Abney, F.R.S., exhibited Photographs of Flames.

Prof. O. Henrici, F.R.S., read a paper "On an Elementary Theory of Planimeters."

Mr. F. W. Hill made a communication "On the Hatchet Planimeter."

Mr. A. Sharp, B.Sc., made a communication on "A new Integrating Apparatus."

Prof. Perry, F.R.S., made a communication on "Magnetic Shielding."

Mr. S. Skinner made a communication "On Clark's Cells."

October 26th, 1894.

Prof. A. W. Rücker, President, in the Chair.

A paper was read, "On the Influence of Temperature on the Specific Heat of Aniline," by E. H. Griffiths, M.A.

November 9th, 1894.

Prof. A. W. Rücker, President, in the Chair.

Messrs. D. Rintoul, R. Appleyard, and W. Barlow were elected Members of the Society.

A paper by Dr. J. Larmor on "The Significance of Wiener's Localization of the Photographic Action of Stationary Light-Waves" was read by Mr. Elder.

Dr. Sydney Young, F.R.S., read a paper on "The Influence of the Relative Volumes of Liquid and Vapour on the Vapour-pressure of a Liquid at Constant Temperature."

Mr. Burke read a paper, "On a Suggestion by Prof. J. J. Thomson in connexion with the Luminescence of Glass due to Kathode Rays."

November 23rd, 1894.

Prof. A. W. Rücker, President, in the Chair.

Mr. J. Burke was elected a Member of the Society.

Mr. Womack read a paper on "A Modification of the Ballistic Galvanometer Method of Determining the Electromagnetic Capacity of a Condenser."

Prof. S. P. Thompson, F.R.S., and Mr. Miles Walker communicated a paper on "Mirrors of Magnetism."

Prof. W. E. Ayrton, F.R.S., and Mr. Mather exhibited a "Student's Apparatus for Verifying Ohm's Law."

Prof. Ayrton also showed an Idiostatic Electrometer.

December 14th, 1894.

Prof. A. W. Rücker, President, in the Chair.

Messrs. G. H. Bryan, J. W. Capstick, T. C. Fitzpatrick, G. F. C. Searle, S. Skinner, W. C. D. Whetham, L. R. Wilberforce, and Major P. MacMahon, R.A., were elected Members of the Society.

Prof. W. E. Ayrton, F.R.S., and Mr. H. C. Haycraft communicated a paper on "Students' Simple Apparatus for Determining the Mechanical Equivalent of Heat."

Prof. W. E. Ayrton and Mr. E. A. Medley communicated a paper on "Tests of Glow-Lamps, and Description of the Measuring Instruments employed."

January 11th, 1895.

(Held by invitation of Professor Carey Foster at the Physical Laboratory of University College.)

Prof. A. W. Rücker, President, in the Chair.

Prof. Ramsay and Miss Dorothy Marshall communicated a paper on "The Measurement of Latent Heats of Vaporization of certain Organic Liquids."

Mr. Eumorfopoulos read a paper on "The Determination of Thermal Conductivity and Emissivity."

Mr. A. W. Porter read a paper on "The Influence of the Dimensions of a Body upon the Thermal Emission from its Surface."

Mr. G. U. Yule communicated a paper on "The Passage of an Oscillator Wave-train through a Plate of Conducting Dielectric."

January 25th, 1895.

Prof. A. W. Rücker, President, in the Chair.

Prof. W. T. A. Emtage was elected a Member of the Society.

Prof. W. E. Ayrton, F.R.S., and Mr. Medley continued the paper (begun Dec. 14th, 1894) on "Tests of Glow-Lamps, and Description of the Measuring Instruments employed."

Prof. Anderson and Mr. J. A. McClelland communicated a paper on "The Temperature of Water at its Maximum Density, and its Coefficient of Expansion in the Neighbourhood of this Temperature."

Annual General Meeting.

February 8th, 1895.

Prof. A. W. Rücker, President, in the Chair.

The following Report of the Council was read by the Chairman : —

Within the year which has elapsed since the last Annual General Meeting very considerable changes have been carried out in the directions indicated in the last Report. The Council has set on foot the monthly issue of a publication which contains Abstracts of important Foreign papers as well as papers which have been submitted directly to the Society.

A valuable list of the chief Memoirs on Physics of Matter, compiled by Mr. R. A. Lehfeldt, has been published by the Society, and a copy of this pamphlet has been sent to every Member.

The place of Meeting has been changed, and it is hoped that, on the whole, Members will find these rooms, in which, by the kind arrangement of the Chemical Society, we are now enabled to meet, rather more convenient and accessible than our old home at South Kensington. In saying this, however, the Council wishes emphatically to express the sense of its Members of the great advantages which, during the youth of this Society, have accrued to it from the kind way in which the rooms at S. Kensington have been placed at its service by the Lords' Committee of the Council on Education.

Tea and coffee are now provided before the Meetings.

Since the last General Meeting there have been fifteen Ordinary Business Meetings. Thirteen of them have been held at the usual place, one under special circumstances at the Institute of the City and Guilds of London, with the permission of Prof. Ayrton, and one at the invitation of Prof. Carey Foster and the Staff of University College, Gower Street, at the newly organized Laboratory of that Institution.

The Members of the Society number now about 370.

A vacancy in the list of Honorary Members has taken place in the death of Herr H. von Helmholtz.

The other deaths in the ranks of the Society, news of which has reached the Council, are those of Mr. A. C. Ranyard, Col. Armstrong, and Mr. Alder Wright.

The following additions have been made to the Library during the past year:—

Newspapers and Magazines:—

Nature.
The Electrical Review.
The Electrician.
Electrical Plant.
Engineering.
Industries and Iron.
The Open Court.
The Philosophical Magazine.
Journal de Physique.
Beiblätter.
Annalen der Physique und Chemie.

Journals of Societies, British:—

Journal of the Society of Arts.
Proceedings of the Birmingham Philosophical Society.
Transactions and Proceedings of the Cambridge Philosophical Society.
Journal of the Camera Club.
Transactions and Proceedings of the Royal Dublin Society.
Journal of the Institute of Electrical Engineers.
Memoirs and Proceedings of the Manchester Literary and Philosophical Society.
Proceedings of the Institute of Mechanical Engineers.
Quarterly Journal of the Royal Meteorological Society.
Proceedings of the Royal Institution.
Proceedings of the Royal Society.

Journals of Societies, Colonial:—

Journal and Proceedings of the Royal Society of New South Wales.
Geological and Natural History Survey of Canada.
Hong Kong Observatory Observations.
Proceedings and Transactions of the Nova Scotia Institute of Natural Science.
Transactions of the Canadian Institute.
Papers of the Mathematical and Physical Society of Toronto.

Journals of Societies, Foreign :—

Proceedings of the Academy of Natural Sciences, Philadelphia.
Journal of the Physical and Chemical Society of Russia.
Verhandlungen, Berlin Physical Society.
Mémoires and Séances of the Physical Society of France.
Journal of the College of Science of Japan.
Séances and Bulletin International de l'Académie des Sciences, Cracovie.
Report of the Ohio Meteorological Bureau.
Technology Quarterly and Proceedings of the Society of Arts, Massachusetts.
Report of the Ohio Weather and Crop Service.
Prace Matematyczno-Fizyczne, Warsaw.

Books, &c. :—

Norwegian North Sea Expedition, xxii.
Bureau International des Poids et Mesures. Travaux et Mémoires, viii.–x.
——. Seizième rapport.
On the Origin of Moss Gold, &c., by A. Liversidge.
Stonyhurst College Observatory Meteorological and Magnetical Observations.
Bombay Magnetical and Meteorological Observations.
Smithsonian Report.
Œuvres de Laplace, Tome x.
Catalogue of Books, Radcliffe Library, Oxford.
A New Story of the Stars, by A. W. Bickerston.
Kew Observatory Report.
The Electric Current, by R. M. Walmsley.
Calendars of University College and Imperial University of Japan.

Hermann Ludwig Ferdinand von Helmholtz, Honorary Member of the Society, was born August 31, 1821, at Potsdam, where his father, Ferdinand Helmholtz, was Professor in the Gymnasium. His mother, Caroline Penn, was of an English family which had emigrated to Prussia. In early years he was delicate, as he tells us in the autobiographical address delivered on the occasion of his seventieth birthday in 1891. His first scientific ideas came to him when a child playing with his wooden blocks. His father used to take him about much in the neighbourhood of Potsdam, and encouraged in him the love of Nature. Hence, he says, his first fragments of Physics had more interest for him than purely geometrical and algebraical studies. He was fascinated by the intellectual mastery over nature given by the logical force of law.

The books on Physics he could read in his father's library were old. "Phlogiston held sway; galvanism had not grown beyond the voltaic pile;" spectacle-glasses afforded a means of experiments with lenses; and the theory of the telescope had more of interest for him than Cicero or Virgil.

But Physics offered no means of livelihood to its student; and so he entered the Army Medical School at Berlin, becoming a pupil of Johannes Müller. In early days he began to consider the nature of Vital Force; the ideas then in favour among physiologists were those of Stahl, and Helmholtz, reflecting on them, was led to see that Stahl's theory "ascribed to every living body the nature of a *perpetuum mobile*." From this grew the Essay "On the Conservation of Force," published in 1847. The questions he placed before himself for solution were two: "What relations must exist between various kinds of natural forces for a perpetual motion to be possible?" and further, "Do these relations exist?" Most of the leading Physicists of his time were disposed to consider the essay a fantastical speculation. Jacobi alone of the older generation seems to have recognized its importance; the younger men received it with enthusiasm.

In 1848 Helmholtz became Assistant in the Anatomical Museum at Berlin, and in the following year Professor of Physiology at Königsberg. In 1856 he was called to Bonn as Professor of Physiology and Anatomy, and in 1859 to Heidelberg as Professor of Physiology; in 1871 he was promoted to Berlin as Professor of Natural Philosophy. About 1887 the wise munificence of Werner von Siemens founded the Berlin Reichsanstalt, and Helmholtz became its first President. It was at Königsberg that he invented

the ophthalmoscope, and "had the joy of being the first to see clearly before him a living human retina." From this time success was assured. His colleagues and the authorities recognized his genius, and for the future he was able to pursue far more freely "the secret impulses of his desire for knowledge."

The work on Physiological Optics followed. Helmholtz had the thorough grasp of Physical principles needed for success in the task which he undertook. A full analysis of the book is far beyond our limits; it deals first with the laws of refraction at spherical surfaces: the theory of colour sensation is then investigated, and its dependence on three variables, in accordance with the ideas of Young, is proved conclusively. Another difficult subject which is carefully treated is that of the motions of the eyes. The question whether there are three primary sensations in the eye corresponding to the three variables of the colour equations remains unsolved. Maxwell's experiments and Helmholtz' work have proved the existence of these three variables, and have shown that any theory which is to explain colour vision must take account of them.

In 1862 was published the first edition of the 'Sensations of Tone as a Physiological Basis for the Theory of Music.' In it the laws of Acoustics are considered from a Physical as well as from a Physiological standpoint, and the relation between the two is shown. The methods of analysing a compound tone into its simple notes are due to him: the work is full of brilliant suggestions and carefully elaborated results.

By this time the Physiologist had become a Physicist and the Physicist a Mathematician. The call to Berlin left him free to develop the ideas which had first shown themselves in the Essay in 1847; and though his interest in Physiology never waned, and in some respects grew stronger, his own work from this time onwards was Physical.

It is impossible to describe here in any detail what that work was. He has written on nearly every branch of our science, and each subject that he has handled has gained from his powerful treatment. "I have been in a position," he writes, "to solve several mathematical physical problems. . . . But the pride which I might have felt about the final result in these cases was considerably lowered by my consciousness that I had only succeeded in solving these problems after many devious ways by the gradually increasing generalization of favourable examples and by a series of fortunate guesses. . . . I had to compare myself with an Alpine

climber who, not knowing his way, ascends slowly and with toil, and is often compelled to retrace his steps because his progress is stopped; sometimes by reasoning, sometimes by accident, he hits upon traces of a fresh path, which again leads him a little further, till at last the summit is reached."

We think of Newton, who describes himself as seeking stones upon a shore, and occasionally finding a pebble more precious than any discovered by the crowd about him; but surely rather, compared with those who struggle to follow where he has led, von Helmholtz was no ordinary Alpine climber, but the Prince of Alpine guides, who, after surveying from a distance an unknown peak, and carefully maturing his plan, leads his party to the top with an unerring skill which is little short of marvellous. He has helped us, however, in our struggles onwards by his assurance that his brilliant suggestions did not come without effort on his part.

"Such moments of fruitful thought," he writes, "were indeed very delightful; but not so the reverse when the redeeming ideas did not come. For weeks or months I was gnawing at such a question until in my mind I was

> 'Like to a beast upon a barren heath,
> Dragged in a circle by an evil spirit,
> While all around are pleasant pastures green.'

And lastly, it was often a sharp attack of headache which released me from this strain, and set me free for other interests."

The Physical papers really began in 1847 with the Essay "On the Conservation of Force"—one could wish the translator had said Energy—and were continued to within a short time of his death last year.

His collected papers were issued in two volumes in 1882 and 1883, each containing about 1000 pages. Of these the first volume and some 200 pages of the second contain physical and mathematical writings.

In his paper "On Vortex Motion" (1858) he—to use Maxwell's words—"establishes principles in pure hydrodynamics which had escaped the penetrative powers of all the mathematicians who preceded him, including Lagrange himself."

The Acoustical papers we have already referred to. Of his Optical work, in addition to the Physiological Optics, the paper on Anomalous Dispersion, published in 1874, is perhaps the most important.

Among the theoretical Electrical papers must be mentioned one

on Electrical Oscillations, published in 1869, containing some of the first measurements on the period of an oscillatory discharge of a condenser through a circuit having self-induction. The most important of Helmholtz' contributions to Electrical theory, however, are contained in his papers on the Theory of Electrodynamic Action. The first of these, dealing with the Equations of Motion of Electricity in Conductors at rest, appeared in Borchardt's Journal, vol. lxxii., in 1870; others followed in 1871 and 1873. A short paper on the Laws of Variable Electric Currents in Solid Conductors, giving a general summary of the results of the full paper in Borchardt's Journal, was read at the meeting of the Naturhistorich-medicinischer Verein at Heidelberg in 1870. Maxwell's great paper on the general equations of the electromagnetic field was published in the 'Philosophical Transactions' for 1865.

In the paper in Borchardt's Journal, Helmholtz obtains equations more general in form than those of Maxwell. He starts by finding the complete expression for the mutual potential energy of two elements of current consistent with Ampère's laws for closed circuits. This expression involves an unknown constant k; the terms containing k disappear whenever the circuits are completely closed; hence the existence of this quantity cannot be proved from experiments on closed circuits. According to Maxwell's theory k is zero; and in this case the only electromagnetic waves which can be propagated are transverse waves. According to Helmholtz' more general form, electrostatic potential is propagated through a dielectric by longitudinal waves which travel with velocity $V/\sqrt{k}$, where V is the velocity of the transverse disturbance. J. J. Thomson's Report on Electrical Theories (Brit. Assoc. Reports, Aberdeen, 1885) gives an account of the efforts made up to that time to determine k and to decide between the two theories, but without success, until in 1888 Helmholtz' most brilliant pupil, Hertz *, astonished the world by his complete verification of Maxwell's theory.

Less than two years ago von Helmholtz contributed his last notable paper to Electromagnetic theory, dealing with Dispersion.

Space compels only the very briefest reference to his work on

* Hertz' own original results did not, it is true, confirm Maxwell's theory completely; the further experiments of Sarasin and de la Rive were needed before all errors were eliminated and the proof made perfect.

Thermodynamics and Chemistry, specially in reference to the actions which take place in a battery, the connexion between its electromotive force and the chemical combinations which occur together with what one may call his electrical theory of chemical action.

Specially interesting, however, to many Members of the Physical Society is his work in connexion with Electrical Units. In his official position as Head of the Reichsanstalt it fell to him to frame the fundamental definitions of the units, and to draft the laws by which those units were legalized in Germany. His visit to England in 1892 was undertaken with a view to securing uniformity in the fundamental definitions throughout the world; and the successful results of the Conference at Edinburgh in that year and the Congress at Chicago in 1893 are in no small measure due to him. Those of us who were then brought into close contact with him were amazed at his marvellous grasp of the small details involved in the various experiments, no less than at the width and clearness of his knowledge.

It is impossible to express in words the debt which Physicists owe to von Helmholtz: his works and the gratitude of his pupils on every hand form his best memorial. The words of the great wise man of old times have already been applied to him; it is fitting that they should again be quoted here—

> "A wise man instructeth his people, and the fruits of his understanding fail not."
>
> "A wise man shall inherit glory among his people, and his name shall be perpetual."

Col. Robert Young Armstrong was of Irish descent, and entered the Royal Engineers in 1858. After serving abroad he returned to England, and was quartered at Chatham, where he was appointed first, Assistant Instructor, and, secondly, Instructor in Electrical Science and Sub-marine Mining, which was then in its infancy. The Service is indebted to many improvements and inventions in this latter branch of defence, and his services were recognized by the Commander-in-Chief by his appointment as Inspector of Submarine Defences, and on his retirement from the Service by his being created a C.B. Col. Armstrong was elected to the Royal Society in 1891. His kindly disposition and unselfishness made him a general favourite with all, and there is no one whose loss could be more deplored by his friends, amongst whom the writer of this notice was proud to be reckoned.

Arthur Cowper Ranyard was born at Swanscombe, in the county of Kent, on June 21, 1845, and, after his father's removal to 13 Hunter Street, Brunswick Square, he was educated at University College School, Gower Street. On leaving the school he entered the adjoining college, where, from the influence of the late Professor De Morgan, whose classes he attended, he acquired a strong love for mathematics and astronomy. Whilst at University College he collaborated with Mr. George De Morgan in founding a "Students' Mathematical Society," which had a most successful career, developing eventually into the present "London Mathematical Society." Mr. Ranyard read the first paper before the new Society, one "On Determinants."

From University College, London, Mr. Ranyard went to Cambridge, entering on residence at Pembroke College in the October term of 1865, and taking his degree in 1868. Three years later he was called to the Bar at Lincoln's Inn; but, whilst adopting the law as his profession, his leisure hours were mostly devoted to science. Thus he joined the Physical Society in its first session, and in 1872 worked with Lord Lindsay (the present Earl of Crawford) in the study of photographic irradiation. Again in 1885 and 1886 the same subject of photography occupied his attention, and he conducted a number of experiments in order to determine the relations between the length of exposure and the intensity of photographic action.

His chief work as an astronomical observer consisted in his observations of the three total solar eclipses of 1872, 1878, and 1882. On the first occasion he was one of the observers sent out by the Joint Committee of the Royal and Royal Astronomical Societies, and was stationed at Villasmunda, near Augusta, Sicily. He was here favoured with much better fortune than awaited most of his colleagues, and was afforded a fine view of the phenomena of totality. The observations assigned to him were polariscopic, and he carried them out with entire success, repeating, with a few additional details, those made by Prazmowski in 1860. His expeditions to America to view the total solar eclipse of 1878, and to Egypt to view that of 1882, were made at his own expense, and he went on his own responsibility. He was contemplating an important work in the department of solar physics on the lines opened up by M. Deslandres and Professor Hale, when the disease which afterwards proved fatal laid its hand upon him. He had had a spectroheliograph constructed for him, and this he lent to Professor

Hale for the latter's expedition to Etna to attempt the photographing of the corona in full sunshine. Mr. Ranyard never had the opportunity of using the instrument himself, and it lay in its cases, not yet unpacked, at the time of his death.

Mr. R. A. Proctor's sudden death in 1887 left the publication of his great work, 'Old and New Astronomy,' in a most unfinished state, and his magazine, 'Knowledge,' without an editor. It was exceedingly typical of Mr. Ranyard's generous spirit that he cheerfully undertook the immense amount of hard work, extending over many months, which the completion of the 'Old and New Astronomy' involved, in order to sustain the reputation of his deceased friend and to assist his family. And difficult as it was to follow a man of Proctor's ability, and to sustain his work upon an equal level, Mr. Ranyard fully succeeded in the task.

Another publication undertaken by Mr. Ranyard was that of Professor De Morgan's little brochure, 'Sir Isaac Newton, his Friend, and his Niece,' to which he supplied a preface. Nor was the reputation of Sir Isaac Newton the only one in which he was interested. He was anxious that the gifted young pioneer of English astronomy should have due honour rendered to him, and was instrumental in having a memorial tablet erected to Jeremiah Horrox in Westminster Abbey.

His connexion with the Royal Astronomical Society began at an unusually early age, he having been elected a Fellow when only 18. His first work in an official capacity was in the preparation and organization of the Eclipse Expedition of 1870; Mr. J. Norman Lockyer being appointed Secretary of the Joint Eclipse Committee of the Royal and Royal Astronomical Societies, and Mr. Ranyard, Assistant-Secretary. He was elected to a seat on the Council in February 1872, and to the Secretaryship in February 1874. This office, which he retained till 1880, he filled with conspicuous industry and courtesy. During his tenure of this office he brought out vol. xli. of the 'Memoirs,' the great "Eclipse" volume. In its inception it was intended to be a report of the eclipses of 1860 and 1870, and at first Mr. Ranyard worked with the assistance and under the oversight of Sir G. B. Airy, then Astronomer Royal. Soon, however, the entire work devolved upon him, and its scope was extended until it embraced the collation and arrangement of the known observations of all eclipses down to 1878. This work, most admirably designed, and most laborious in execution, was not published until 1879.

Early in the summer of 1894 a mysterious loss of weight and other disquieting symptoms began to make themselves apparent. These were eventually traced to the formation of an internal cancer which all but precluded the possibility of nourishment entering the system. After a long and most trying illness, borne with great patience and hopefulness, he died on December 14, 1894, whilst the meeting of the Royal Astronomical Society, in the welfare of which he took from first to last so keen an interest, was in session. Mr. Ranyard was not married, and none of his immediate relatives survived him. But his purity of motive, his generous spirit, his freedom from self-seeking, and his unfailing courtesy of manner, endeared him to a large circle of friends who will keenly feel his loss. Amongst these will be numbered not a few of the most distinguished foreign astronomers, Mr. Ranyard having been an excellent linguist, and frequently visiting the Continent.

These details of Mr. Ranyard's career have been kindly furnished by the Royal Astronomical Society.

The adoption of the Report was proposed and passed unanimously.

The election of Officers and other Members of Council then took place, the new Council being constituted as follows :—

President.—Capt. W. de W. Abney, R.E., C.B., D.C.L., F.R.S.

Vice-Presidents who have filled the Office of President.—Dr. J. H. Gladstone, F.R.S.; Prof. G. C. Foster, F.R.S.; Prof. W. G. Adams, M.A., F.R.S.; Lord Kelvin, D.C.L., LL.D., P.R.S.; Prof. R. B. Clifton, M.A., F.R.S.; Prof. A. W. Reinold, M.A., F.R.S.; Prof. W. E. Ayrton, F.R.S.; Prof. G. F. Fitzgerald, M.A., F.R.S.; Prof. A. W. Rücker, M.A., F.R.S.

Vice-Presidents.—Walter Baily, M.A.; Major-General E. R. Festing, R.E., F.R.S.; Prof. J. Perry, D.Sc., F.R.S.; G. Johnstone Stoney, M.A., F.R.S.

Secretaries.—T. H. Blakesley., M.A., M.Inst.C.E.; H. M. Elder, M.A.

Treasurer.—Dr. E. Atkinson.

Demonstrator and Librarian.—C. Vernon Boys, F.R.S.

Other Members of Council.—Shelford Bidwell, M.A., LL.B., F.R.S.; Prof. W. Crookes, F.R.S.; L. Fletcher, M.A., F.R.S.; R. T. Glazebrook, M.A., F.R.S., G. Griffith, M.A.; Prof. O. Henrici, Ph.D., F.R.S.; Prof. G. M. Minchin, M.A.; J. Swinburne; Prof. S. P. Thompson, D.Sc., F.R.S.; Prof. S. Young, D.Sc., F.R.S.

Votes of thanks were passed to the Lords' Committee of the Council on Education; to the Officers; and to the Auditors.

The following Resolution, of which notice had been duly given, was moved from the Chair and carried:—

"That the Articles of Association of the Society be altered in the manner following:—

"(*a*) In Article 11, after the word 'member' to insert the words 'except as hereinafter appointed.'

"(*b*) After Article 12, to insert 'Article 12*a*. The Council may at their discretion recommend to the Society for election as a member any person not qualified according to Article 11, when in the opinion of at least three fourths of those present at the Council Meeting circumstances render it desirable to do so.'"

The Treasurer in Account with the Physical Society, from January 1st, 1894, to December 31st, 1894.

Dr.	£	*s.*	*d.*	£	*s.*	*d.*
Balance in the Bank, January 1, 1894				239	9	11
Dividends, less Income Tax:—						
£400 Furness 4 per cent. Debenture Stock	15	10	4			
£200 Lancaster Corporation Stock (3 half-years)	11	13	0			
£1000 Midland 4 per cent. Preference Stock	38	15	10			
£254$\frac{2}{5}$ New South Wales 3$\frac{1}{2}$ per cent. Stock	8	12	4			
£200 Metropolitan Board of Works Stock	6	15	8			
				81	7	2
Taylor and Francis:—						
Sales, less Commission				22	4	11
Entrance-Fees	10	0	0			
Life Compositions	90	0	0			
Subscriptions for 1888	1	0	0			
„ 1889	1	0	0			
„ 1890	1	0	0			
„ 1891	2	0	0			
„ 1892	4	0	0			
„ 1893	8	0	0			
„ 1894	109	0	0			
„ 1895	3	0	0			
				229	0	0
Balance due to the Treasurer				4	18	6
				£577	0	6

Cr.	£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1, 1894. Balance due to the Treasurer				2	18	0
Taylor and Francis:—						
Proceedings, vol. xii. (parts 3 and 4), and vol. xiii. (part 1)	138	14	6			
Members' separate copies	22	14	6			
Miscellaneous printing	40	5	10			
Postage and sending out	13	17	6			
Lehfeldt's Bibliography	24	18	1			
				240	10	5
Williams and Norgate				6	4	4
Chapman: Attendance and petty cash				5	14	1
Reporting:—						
Dr. Mather	17	12	11			
Dr. Burton	4	14	6			
				22	7	5
Binding:—						
Mr. Hovey	3	7	1			
Mr. Gyde	1	19	3			
				5	6	4
Chemical Society:—						
Hire of Rooms				6	16	6
Lancaster Corporation Stock purchased				3	6	8
Expenses of Tea				5	0	0
Petty Cash:—						
Dr. Atkinson	1	8	0			
Mr. Elder	3	11	0			
Mr. Blakesley (2 years)	1	12	11			
				6	11	11
Balance in the Bank, Dec. 31, 1894				272	4	10
				£577	0	6

EDMUND ATKINSON, *Treasurer.*

Examined and found to agree with the vouchers, books, and accounts.

RICHARD INWARDS,
A. P. TROTTER, } *Auditors.*

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, DECEMBER 31, 1894.

ASSETS.	£	s.	d.
Balance at Bank	272	4	10
Subscriptions due, estimated	34	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0
£1000 Midland 4 per cent. Preference Stock	1194	0	0
£200 Metropolitan Board of Works 3½ per cent. Stock	217	0	0
£270 Lancaster Corporation 3 per cent. Stock	227	6	8
£254⅔ New South Wales 3½ per cent. Inscribed Stock	249	7	5
	£2649	18	11

LIABILITIES.	£	s.	d.
Subscriptions for 1895 in advance	3	0	0
Balance due to the Treasurer	4	18	6
Balance	2642	0	5
	£2649	18	11

EDMUND ATKINSON, *Treasurer.*

Securities examined and found correct. For the New South Wales 3½ per cent. Stock no certificate is given. The Bankers have duly received the last dividend for October 1894.

RICHARD INWARDS,
A. P. TROTTER, } *Auditors.*

PHYSICAL SOCIETY OF LONDON.

ABSTRACTS OF PHYSICAL PAPERS

FROM FOREIGN SOURCES.

MAY 1895.

GENERAL PHYSICS.

342. *Equilibrium of an Indefinite Elastic Body limited by a Plane.* **G. Lauricella.** (Nuov. Cim. 36. pp. 173-183, 1894.)—Let the bounding planes be the plane of yz, the body being on the positive side of it. Let $x_1 y_1 z_1$ be any point within S, and let r be the distance of any point from it, or

$$r^2 = (x-x_1)^2+(y-y_1)^2+(z-z_1)^2.$$

Let $-x_1 y_1 z_1$ be the corresponding point, or image of $x_1 y_1 z_1$, on the opposite side of the plane of yz. And let

$$r_1^2 = (x+x_1)^2+(y-y_1)^2+(z-z_1)^2.$$

Then on the plane $x=0$ the following relations hold, viz.,

$$\frac{dr_1}{dx} = -\frac{dr}{dx}, \qquad \frac{dr_1}{dx_1} = \frac{dr}{dx_1};$$

and so for $y\,y_1$, $z\,z_1$,

$$\frac{dr_1}{dy} = \frac{dr}{dy}, \qquad \frac{dr_1}{dz} = \frac{dr}{dz}, \qquad \frac{dr_1}{dy_1} = \frac{dr}{dy_1}, \qquad \frac{dr_1}{dz_1} = \frac{dr}{dz_1}.$$

I. Let now the forces ξ, η, ζ acting at any point in the body be given, and let the displacements u, v, w on the surface or plane of yz be given. The following three sets of functions are integrals of the equations of equilibrium, and are finite at all points within S :—

$$\begin{cases} a_1 = -\dfrac{1}{r_1} - \dfrac{\alpha}{2}\dfrac{d^2 r_1}{dx^2} - \alpha x_1 \mathrm{M}x \dfrac{d^2}{dx^2}\dfrac{1}{r_1}, \\ b_1 = -\dfrac{\alpha}{2}\dfrac{d^2 r_1}{dx\,dy} + \alpha x_1 \dfrac{d}{dy}\dfrac{1}{r_1} - \alpha x_1 \mathrm{M}x \dfrac{d^2}{dx\,dy}\dfrac{1}{r_1}, \\ c_1 = -\dfrac{\alpha}{2}\dfrac{d^2 r_1}{dx\,dz} + \alpha x_1 \dfrac{d}{dz}\dfrac{1}{r_1} - \alpha x_1 \mathrm{M}x \dfrac{d^2}{dx\,dz}\dfrac{1}{r_1}; \end{cases}$$

$$\begin{cases} a_2 = -\dfrac{\alpha}{2}\dfrac{d^2 r_1}{dy\,dx} + \alpha x_1 \dfrac{d}{dy}\dfrac{1}{r_1} + \alpha x_1 \mathrm{M}x \dfrac{d^2}{dy\,dx}\dfrac{1}{r_1}, \\ b_2 = -\dfrac{1}{r_1} - \dfrac{\alpha}{2}\dfrac{d^2 r_1}{dy^2} + \alpha x_1 \mathrm{M}x \dfrac{d^2}{dy^2}\dfrac{1}{r_1}, \\ c_2 = -\dfrac{\alpha}{2}\dfrac{d^2 r_1}{dy\,dz} + \alpha x_1 \mathrm{M}x \dfrac{d^2}{dy\,dz}\dfrac{1}{r_1}; \end{cases}$$

$$\begin{cases} a_3 = -\dfrac{\alpha}{2}\dfrac{d^2 r_1}{dx\,dz} + \alpha x_1 \dfrac{d}{dz}\dfrac{1}{r_1} + \alpha x_1 \mathrm{M}x \dfrac{d^2}{dx\,dz}\dfrac{1}{r_1}, \\ b_3 = -\dfrac{\alpha}{2}\dfrac{d^2 r_1}{dy\,dz} + \alpha x_1 \mathrm{M}x \dfrac{dz}{dy\,dz}\dfrac{1}{r_1}, \\ c_3 = -\dfrac{1}{r_1} - \dfrac{\alpha}{2}\dfrac{d^2 r_1}{dz^2} + \alpha x_1 \mathrm{M}x \dfrac{d^2}{dz^2}\dfrac{1}{r_1}; \end{cases}$$

in which α and M are constants depending on the nature of the body. On the plane $x=0$, since then $r=r_1$, these assume values equal and opposite to

$$u_1 = \frac{1}{r} + \frac{\alpha}{2}\frac{d^2 r}{dx^2}, \qquad v_1 = \frac{\alpha}{2}\frac{d^2 r}{dx\,dy}, \qquad w_1 = \frac{\alpha}{2}\frac{d^2 r}{dx\,dz};$$

$$u_2 = \frac{\alpha}{2}\frac{d^2 r}{dx\,dy}, \qquad v_2 = \frac{1}{r} + \frac{\alpha}{2}\frac{d^2 r}{dy^2}, \qquad w_2 = \frac{\alpha}{2}\frac{d^2 r}{dy\,dz};$$

$$u_3 = \frac{\alpha}{2}\frac{d^2 r}{dx\,dz}, \qquad v_3 = \frac{\alpha}{2}\frac{d^2 r}{dy\,dz}, \qquad w_3 = \frac{1}{r} + \frac{\alpha}{2}\frac{d^2 r}{dz^2};$$

and these latter are also integrals of the equations of equilibrium. Therefore also a_1+u_1, b_1+v_1, c_1+w_1, &c. are integrals of these equations.

Let $\mathrm{A}_1+\mathrm{X}_1$, $\mathrm{B}_1+\mathrm{Y}_1$, $\mathrm{C}_1+\mathrm{Z}_1$ be the superficial forces which, applied on the plane $x=0$, would maintain the body in equilibrium with the displacements a_1+u_1, b_1+v_1, c_1+w_1. Let A_2, X_2, &c. have the corresponding signification for the displacements a_2+u_2 &c. We can now apply the theorem of Betti, using a_1+u_1 &c. as auxiliary integrals. If $u_0\,v_0\,w_0$ be the displacements at $x_1\,y_1\,z_1$, this gives

$$-4\pi \mathrm{L}u_0 = -\iint\{(\mathrm{A}_1+\mathrm{X}_1)u + (\mathrm{B}_1+\mathrm{Y}_1)v + (\mathrm{C}_1+\mathrm{Z}_1)w\}\,d\sigma$$
$$+\int\int\int\rho\{\xi(u_1+a_1)+\eta(v_1+b_1)+\zeta(w_1+c_1)\}\,dx\,dy\,dz,$$

in which ρ is the density and $u\ v\ w$ the given displacements on the plane $x=0$. Or, if $\xi=\eta=\zeta=0$,

$$-4\pi \mathrm{L}u_0=-\iint\{(\mathrm{A}_1+\mathrm{X}_1)u+(\mathrm{B}_1+\mathrm{Y}_1)v+(\mathrm{C}_1+\mathrm{Z}_1)w\}d\sigma.$$

Similarly, using a_2+u_2 &c.,

$$-4\pi \mathrm{L}v_0=-\iint\{(\mathrm{A}_2+\mathrm{X}_2)u+(\mathrm{B}_2+\mathrm{Y}_2)v+(\mathrm{C}_2+\mathrm{Z}_2)w\}d\sigma,$$

$$-4\pi \mathrm{L}w_0=-\iint\{(\mathrm{A}_3+\mathrm{X}_3)u+(\mathrm{B}_3+\mathrm{Y}_3)v+(\mathrm{C}_3+\mathrm{Z}_3)w\}d\sigma.$$

II. Let X, the tension in direction x, and $v\ w$, the components of displacements in directions y and z, be known at each point of the plane $x=0$. In this case the following auxiliary integrals are used, viz.

$$u_1\ v_1\ w_1, \qquad u_2\ v_2\ w_2, \qquad u_3\ v_3\ w_3,$$

with the same meaning as above; and

$$a_1=-\frac{1}{r_1}-\frac{\alpha}{2}\frac{dr_1}{dx^2}, \qquad b_1=-\frac{\alpha}{2}\frac{d^2r_1}{dx\,dy}, \qquad c_1=-\frac{\alpha}{2}\frac{d^2r_1}{dx\,dz},$$

$$a_2=-\frac{\alpha}{2}\frac{d^2r_1}{dx\,dy}, \qquad b_2=-\frac{1}{r_1}-\frac{\alpha}{2}\frac{d^2r_1}{dy^2}, \qquad c_2=-\frac{\alpha}{2}\frac{d^2r_1}{dy\,dz},$$

$$a_3=-\frac{\alpha}{2}\frac{d^2r_1}{dx\,dz}, \qquad b_3=-\frac{\alpha}{2}\frac{d^2r_1}{dy\,dz}, \qquad c_3=-\frac{1}{r_1}-\frac{\alpha}{2}\frac{d^2r_1}{dz^2},$$

$$a_{11}=-\frac{da_1}{dx}=\frac{d}{dx}\frac{1}{r_1}+\frac{\alpha}{2}\frac{d^3r_1}{dx^3},$$

$$b_{11}=-\frac{db_1}{dx}=\frac{\alpha}{2}\frac{d^3r_1}{dx^2\,dy},$$

$$c_{11}=-\frac{dc_1}{dx}=\frac{\alpha}{2}\frac{d^3r_1}{dx^2\,dz},$$

$$a_{12}=\frac{da_1}{dy}+\frac{da_2}{dx}=-\frac{d}{dy}\frac{1}{r_1}-\alpha\frac{d^3r_1}{dx^2\,dy},$$

$$b_{12}=\frac{db_1}{dy}+\frac{db_2}{dx}=-\frac{d\frac{1}{r_1}}{dx}-\alpha\frac{d^3r_1}{dx\,dy^2},$$

$$c_{12}=\frac{dc_1}{dy}+\frac{dc_2}{dx}=-\alpha\frac{d^3r_1}{dx\,dy\,dz},$$

$$a_{13}=\frac{da_1}{dz}+\frac{da_2}{dx}=-\frac{d}{dz}\frac{1}{r_1}-\alpha\frac{d^3r_1}{dx^2\,dz},$$

&c.

Then also let

$$X_{12}=\frac{dX_1}{dy}+\frac{dX_2}{dx}, \quad A_{12}=\frac{dA_1}{dy}+\frac{dA_2}{dx}, \quad \&c.;$$

and therefore

$$X_{12}+A_{12}=0,$$

$$Y_{12}=\frac{dy_1}{dy}+\frac{dy_2}{dx}, \quad B_{12}=\frac{dB_1}{dy}+\frac{dB_2}{dx}, \quad \text{and} \quad Y_{12}+B_{12}=2Y_{12}.$$

It is now found that the auxiliary displacements in x on the plane $x=0$ are the same for the systems u_2 &c. and a_2 &c.; or

$$u_2+a_2=2u_2,$$

but

$$A_2+X_2=0.$$

On the other hand,

$$v_2+b_2=0, \qquad w_2+c_2=0,$$

but

$$B_2+Y_2=2Y_2, \quad C_2+Z_2=2Z_2;$$

and similar relations hold for u_3+a_3, &c. So, applying Betti's theorem, we obtain

$$-4\pi L v_0 = \iint \{2Xu_2-2Y_2v-2Z_2w\}d\sigma,$$

$$-4\pi L w_0 = \iint \{2Xu_3-2Y_3v-2Z_3w\}d\sigma,$$

$$-4\pi L \frac{du_0}{dx} = \iint \left\{ 2X\frac{du_1}{dx}-2\frac{dY_1}{dx}v-2\frac{dZ_1}{dx}w \right\} d\sigma,$$

$$-4\pi L \left(\frac{du_0}{dy}+\frac{dv_0}{dx}\right) = \iint \left\{ 2X\left(\frac{du_1}{dy}+\frac{dv_1}{dx}\right)-2v\left(\frac{dY_1}{dy}+\frac{dY_2}{dx}\right) -2w\left(\frac{dZ_1}{dy}+\frac{dZ_2}{dx}\right)\right\} d\sigma,$$

$$-4\pi L \left(\frac{du_0}{dz}+\frac{dw_0}{dx}\right) = \iint \left\{ 2X\left(\frac{du_1}{dz}+\frac{dw_1}{dx}\right)-2v\left(\frac{dY_1}{dz}+\frac{dY_3}{dx}\right) -2w\left(\frac{dZ_1}{dz}+\frac{dZ_3}{dx}\right)\right\} d\sigma.$$

and

$$u_0=\int\left(\frac{du}{dx}dx+\frac{du}{dy}dy+\frac{du}{dz}dz\right),$$

giving the displacements $u_0\,v_0\,w_0$ at $x_1\,y_1\,z_1$.

In the same manner the two other cases are treated, namely :—III., in which on the plane $x=0$ the displacements in x and the tensions in y and z are given; and IV., in which on the plane $x=0$ the three components of tension are given. S. H. B.

343. *Somigliana's Paper on Elasticity.* **G. Lauricella.** (Nuov. Cim. 36. pp. 225–235, 1894.)—The writer considers Somigliana's surface integrals

$$\iint (Xu' + Yv' + Zw')\, dS, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\iint (X'u + Y'v + Z'w)\, dS, \quad . \quad , \quad . \quad . \quad . \quad . \quad (3)$$

in which X Y Z are the components of given forces acting on the surface of an elastic body, $u\ v\ w$ the displacements; also $u'\ v'\ w'$ are a set of auxiliary displacements which satisfy the conditions of equilibrium within the body, and which would be maintained in equilibrium under the action of the superficial forces X' Y' Z'. It is assumed that the curvature of the surface S is finite.

Consider first the integral

$$\iint (Xu' + Yv' + Zw')\, dS. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let $x_1\ y_1\ z_1$ be the coordinates of a point P_1 within the body, $x\ y\ z$ those of any point on the surface. Let

$$r^2 = (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2.$$

Then each of the three systems of displacements,

$$u_1' = \frac{1}{r} + \frac{\alpha}{2}\frac{d^2 r}{dx^2}, \qquad u_2' = \frac{\alpha}{2}\frac{d^2 r}{dx\, dy}, \qquad u_3' = \frac{\alpha}{2}\frac{d^2 r}{dx\, dz},$$

$$v_1' = \frac{\alpha}{2}\frac{d^2 r}{dy\, dx}, \qquad v_2' = \frac{1}{r} + \frac{\alpha}{2}\frac{d^2 r}{dy^2}, \qquad v_3' = \frac{\alpha}{2}\frac{d^2 r}{dy\, dz},$$

$$w_1' = \frac{\alpha}{2}\frac{d^2 r}{dz\, dx}, \qquad w_2' = \frac{\alpha}{2}\frac{d^2 r}{dz\, dy}, \qquad w_3' = \frac{1}{r} + \frac{\alpha}{2}\frac{d^2 r}{dz^2},$$

satisfies the conditions of equilibrium at P_1, and each of these systems can be maintained in equilibrium by suitable superficial forces.

Therefore also the system

$$\iint (Xu_1' + Yv_2' + Zw_3')\, dS$$

satisfies the conditions at P_1, and this system can be maintained by suitable superficial forces.

These forces will be found as follows:—Let P be a point on the surface such that PP_1 is normal at P, and let P_2 be an external point in the same normal. Let now P_1 be made to move along the normal P_1P till it coincides with P. Then the components are finite throughout. They are:—

$$U_s = L\iint X\frac{du_1}{d\nu} + \&c.\, dS + (L+K)\iint X\left(\frac{dx_1}{d\nu}\frac{du_1'}{dx_1} + \frac{dy_1}{d\nu}\frac{du_2'}{dx_1} + \frac{dz_1}{d\nu}\frac{du_3'}{dx_1}\right) dS,$$

in which L and L+K are constants depending on the nature of the body, and they with α satisfy the condition

$$\alpha = -\frac{L+K}{2L+K},$$

and $d\nu$ is an element of the normal at P measured inwards.

If $d\nu'$ be an element of the same normal measured outwards, the component of force outside of the surface will be

$$U_s' = L\iint X\left(\frac{du_1}{d\nu'} + \&c.\right)dS + (L+K)\iint X\left(\frac{dx_1}{d\nu'}\frac{du_1'}{dx_1} + \frac{dy_1}{d\nu'}\frac{du_2'}{dx_1} + \frac{dz_1}{d\nu'}\frac{du_2'}{dz_1}\right)dS.$$

But it is shown that when P approaches the surface, the sum of the two expressions above given for U_s and U_s' is $-4\pi LX'$, where X' is the value of X at the point P on the surface; that is,

$$U_s + U_s' = -4\pi LX'.$$

Similarly for the other components,

$$V_s + V_s' = -4\pi LY',$$

$$W_s + W_s' = -4\pi LZ',$$

equations analogous to those which obtain on a surface charged with electricity.

To pass now to the other surface-integral of Somigliana,

$$\iint (X'u + Y'v + Z'w)\,dS. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this expression $X'\,Y'\,Z'$ are the superficial forces which would maintain the body in equilibrium with the displacements $u_1'\,v_1'\,w_1'$ or $u_2'\,v_2'\,w_2'$ or $u_3'\,v_3'\,w_3'$ above given, and $u_1'\,v_1'\,w_1'$ &c. are functions of r, the distance from any point on the surface to the point $P_1(x_1\,y_1\,z_1)$ within, or P_2 without, S. Also $u\,v\,w$ are given arbitrary displacements on S, and $u_0\,v_0\,w_0$ the values which they have at the point P. It is then found that

$$\lim_{P_1P'=0}\int X'u\,dS - \lim_{P_2P'=0}\int X'u\,dS = 4\pi Lu_0,$$

with two corresponding equations for v_0 and w_0.

Finally we form the system of tensions corresponding to displacements of the same form with the integrals (3) of Somigliana, with the result that they are found to be continuous throughout all space. S. H. B.

344. *Deformation of an Elastic Isotropic Circular Area for given Displacements of Points on its Circumference.* **G. Lauricella.** (Nuov. Cim. 4. 1. pp. 87–96, 1895.)—Suppose an elastic medium capable of deformation in two dimensions. We have then to do only with a plane, say the plane of $x\,y$.

The equations of equilibrium are then

$$\left.\begin{aligned} 4\Delta^2 u+(\mathrm{L}+\mathrm{K})\frac{d}{dx}\left(\frac{du}{dx}+\frac{dv}{dy}\right)=0 \\ 4\Delta^2 v+(\mathrm{L}+\mathrm{K})\frac{d}{dy}\left(\frac{du}{dy}+\frac{dv}{dy}\right)=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

where $u\ v$ are the displacements in x and y and

$$\Delta^2=\frac{d^2}{dx^2}+\frac{d^2}{dy^2}.$$

The problem is, Given arbitrarily two single-valued finite and continuous functions, $u(\mu)$, $v(\mu)$, of points μ on the circumference of a circle in the plane of xy, to find two functions R and S which at all points within the circle satisfy (1), and which, as a point n $(x_1 y_1)$ within the circle approaches infinitely near to a point μ on the circumference, tend to become $u(\mu)$, $v(\mu)$ respectively.

Let $$r^2=(x-x_1)^2+(y-y_1)^2,$$

$$\mathrm{M}=-\frac{\mathrm{L}+\mathrm{K}}{3\mathrm{L}+\mathrm{K}}.$$

Then we have three integrals of (1), viz. :—

$$u_1=\log r+\mathrm{M}\left(\frac{dr}{dx}\right)^2, \qquad v_1=\mathrm{M}\frac{dr}{dx}\frac{dr}{dy};$$

$$u_2=\frac{d\log r}{dx}, \qquad v_2=\frac{d\log r}{dy};$$

$$u_3=\frac{d\log r}{dy}, \qquad v_3=-\frac{d\log r}{dx}.$$

Let σ denote any finite area, s its contour, ν an element of the normal to s. Then from the above three integrals are derived other three, viz. :—

$$u_1'=\frac{du_1}{d\nu}=\frac{d\log r}{d\nu}+\mathrm{M}\left\{2\frac{d\log r}{dx}\frac{dx}{d\nu}-2\left(\frac{dr}{dx}\right)^2\frac{d\log r}{d\nu}\right\};$$

$$v_1'=\frac{dv_1}{d\nu}=\mathrm{M}\left\{\frac{d\log r}{dx}\frac{dy}{d\nu}+\frac{d\log r}{dy}\frac{dx}{d\nu}-2\frac{dr}{dx}\frac{dr}{dy}\frac{d\log r}{d\nu}\right\};$$

$$u_2'=\frac{d\log r}{dx}\frac{dx}{d\nu}; \qquad v_2'=\frac{d\log r}{dy}\frac{dx}{d\nu};$$

$$u_3' = \frac{d\log r}{d\nu} - \frac{d\log r}{dx}\frac{dx}{d\nu}; \qquad v_3' = -\frac{d\log r}{dx}\frac{dy}{d\nu}$$

and from these two others, viz.:

$$a = u_1' - \mathrm{M}u_2' + \mathrm{M}u_3';$$
$$b = v_1' - \mathrm{M}v_2' + \mathrm{M}v_3';$$

or
$$a = (1+\mathrm{M})\frac{d\log r}{d\nu} - 2\,\mathrm{M}\left(\frac{dr}{dx}\right)^2\frac{d\log r}{d\nu};$$
$$b = -2\mathrm{M}\,\frac{dr}{dx}\frac{dr}{dy}\frac{d\log r}{d\nu}.$$

Now, by Green's theorem in two dimensions when the point from which r is measured is within σ,

$$\int\frac{d\log r}{d\nu}\,ds = 2\pi, \qquad \int 2\left(\frac{dr}{dx}\right)^2\frac{d\log r}{d\nu}\,ds = 2\pi,$$
$$\int\frac{dr}{dx}\frac{dr}{dy}\frac{d\log r}{dr}\,ds = 0.$$

But if the point $x_1\,y_1$ be on the contour s,

$$\int\frac{d\log r}{d\nu}\,ds = \pi, \qquad \int 2\left(\frac{dr}{dx}\right)^2\frac{d\log r}{d\nu} = \pi,$$
$$\int\frac{dr}{dx}\frac{dr}{dy}\frac{d\log r}{d\nu} = 0.$$

Therefore, for all points within σ,

$$\int a\,ds = 2\pi, \quad \int b\,ds = 0,$$

and for all points on s,

$$\int a'\,ds = \pi, \quad \int b'\,ds = 0.$$

Let $u\,(\mu)$ be any finite and continuous function of the coordinates of a point μ on s, n any point within σ.

Then evidently

$$\mathrm{U}\,(n) = \frac{1}{\pi}\int u(\mu)a\,ds$$

and

$$\mathrm{V}\,(n) = \frac{1}{\pi}\int u(\mu)b\,ds$$

are two functions, single-valued, finite, and continuous, which satisfy (1) at all points within σ.

It is then shown that

$$\lim_{n=\mu}\mathrm{U}(n) = \mathrm{U}_\mu + u_\mu,$$
$$\lim_{n=\mu}\mathrm{V}(n) = \mathrm{V}_\mu.$$

U (n) and V(n) are not then the solution of the problem, because although they satisfy (1) within σ, they do not become u (μ) and v (μ) on the circumference.

Let the area σ now become a circle of radius R. Let r be the distance from a point x_1 y_1 within the circle, and let it be denoted by r_0 where x_1 y_1 is on the circumference. Then it is found that the two functions

$$\mathrm{M}(n) = -\frac{1}{\pi \mathrm{R}} \int u(\mu) \log r_0 ds - \frac{1}{\pi \mathrm{R}} \mathrm{U}(\mu);$$

$$\mathrm{N}(n) = -\frac{1}{\pi \mathrm{R}} \mathrm{V}(\mu)$$

satisfy (1) within the circle; and therefore the two functions

$$\left.\begin{matrix} \mathrm{U}(n)+\mathrm{M}(n) \\ \mathrm{V}(n)+\mathrm{N}(n) \end{matrix}\right\} \text{satisfy (1) and become respectively}$$

$$\left.\begin{matrix} -\frac{1}{\pi \mathrm{R}} \int u(\mu) \log r_0 ds + u\mu \\ \text{and zero.} \end{matrix}\right\} \text{on S}$$

It is now required to find two functions $\xi(n)$ and $\eta(n)$ of the coordinates of u which shall satisfy (1) and shall be such that on S, $\xi = \log r_0$, $\eta = 0$.

Let $\rho^2 = x_1^2 + y_1^2$. Let ϕ be Green's function for the circular area. Then the required functions $\xi\,\eta$ are found in the form

$$\xi = \phi - \frac{\mathrm{L}+\mathrm{K}}{2(\mathrm{L}-\mathrm{K})} \frac{\mathrm{R}^2-\rho^2}{\rho} \int_0^\rho \frac{d^2\phi}{dx_1^2} d\rho,$$

$$\eta = -\frac{\mathrm{L}+\mathrm{K}}{2(\mathrm{L}-\mathrm{K})} \frac{\mathrm{R}^2-\rho^2}{\rho} \int_0^\rho \frac{d^2\phi}{dx_1 dy_1} d\rho;$$

and therefore, if

$$\xi_1(n) = \frac{1}{\pi \mathrm{R}} \int u(\mu) \xi ds, \quad \eta_1(n) = \frac{1}{\pi \mathrm{R}} \int u(\mu) \eta ds.$$

Let

$$\mathrm{Q}_1(n) = \mathrm{U}(n) + \mathrm{M}(n) + \xi(n),$$
$$\mathrm{S}_1(n) = \mathrm{V}(n) + \mathrm{N}(n) + \eta(n).$$

Then the functions $\mathrm{Q}_1(n)$, $\mathrm{S}_1(n)$ satisfy (1), and have respectively the values $u(\mu)$ and zero on S. In the same manner can be found two functions $\mathrm{Q}_2(n)$ and $\mathrm{S}_2(n)$ which satisfy (1), and have respectively the values zero and $v(\mu)$ on S; and therefore $\mathrm{Q}_1(n) + \mathrm{Q}_2(n)$ and $\mathrm{S}_1(n) + \mathrm{S}_2(n)$ satisfy (1), and have respectively the values $u(\mu)$ and $v(\mu)$ on S. They are the solution of the problem.

S. H. B.

345. *Astronomical Time Measurements Free from Personal Equation.* **G. Lippmann.** (C. R. 120. pp. 404–407, 1895.)—The best methods for determining the personal equation show that it varies with the same individual, and that a residual error of 0·1 sec. always remains. This means an uncertainty of 1″·5 of arc for equatorial stars. To eliminate the personal equation, M. Lippmann proposes two different methods. The first is photographic. It consists in producing, upon a sensitive plate receiving the images of stars whose position is to be measured, a series of lines representing circles of right ascension from second to second of time. The distances of the stars from these time circles can then be measured at leisure. A fixed narrow illuminated slit is placed in front of the escape-wheel of a clock, so that it is only illuminated for a very short time at the commencement of each second, the instant chosen being that at which the escape wheel is at the middle of its swing. The light from the slit is reflected into a photographic telescope by means of a plain glass mirror, through which the stars may also be seen. When the beam is in the plane of the meridian, the star-chart thus produced shows the celestial vault divided into successive portions, each representing one second of time. The positions of the stars are then determined from the negative by micrometer measurements. The second method is independent of photography, and consists in rendering the time circles permanently visible in the field of a telescope. An opaque disc provided with equidistant vertical lines illuminated by a lamp is placed in the focal plane of a collimating lens. The rays are made parallel, and are reflected into the observing instrument by a plane-parallel transparent mirror. The vertical lines then appear projected on the sky. The opaque disc is displaced horizontally by a micrometer screw driven by clock-work at the rate of one revolution per second. The apparatus is adjusted once for all, so that the distance between two lines is equal to the pitch of the screw; that this distance is seen from the centre of the collimating lens under an angle of 1 sec. of time; and that at the commencement of each second a vertical division is in the plane of the meridian. A seconds pendulum is so arranged that at the middle of its swing a line of light is thrown upon the centre of the field. The zero of the micrometer screw is placed upon this line. Then the star is bisected by one of the vertical lines, and the reading on the micrometer head indicates its distance from the nearest time circle. E. E. F.

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346. *The Animal as a Prime Mover.* **R. H. Thurston.** (J. Frank. Inst. no. 831. pp. 161–177, Mar. 1895.)—In concluding his series of papers upon the energy transformations of animal life, the author discusses the best known facts relating to the electrical and luminous properties of certain organisms. An attempt is made to establish *vitality* as an independent form of energy, but the arguments are not altogether convincing. The rhythmic action of the

heart; the voluntary movement of the animal frame; the entrapping of its victims by the sensitive plant; the motions of the bacteria; the changes of the amœba and the protoplasmic cell, are put forward as "exemplifications of the inherent residence of motor-energy under conditions which involve entire absence of all the machinery of thermo-dynamic, or electro-dynamic, motors of any sort as yet familiar to science;" and from this the author deduces that vital energy is independent of the familiar physical energies and forces. The *psychical* actions, "including consciousness, intellection, and emotion," in so far as they correspond to the transformation of energy, are shown to have a legitimate place in physical research. Finally, attention is called to the break which occurs between the well-known chemical energies supplied to the various systems as food, and the resulting mechanical energies; the intermediate processes of transformation are, at present, uninvestigated, it is only known that they are extremely efficient, and various ways are suggested in which, by studying these operations, a more economical method of obtaining mechanical energy may be possible. R. A.

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347. *Scale-pan Reverser.* **A. Classen.** (Zeit. Instrumk. 15. pp. 101–104, 1895.)—For the finer purposes of exact weighing, and especially for absolute measurements, it is required to interchange the contents of the two scale-pans of a balance, without altering the symmetry, or disturbing the adjustment of the beam. The apparatus here described enables these reversals to be made, without opening the balance-case. In place of the ordinary scale-pans the inventor uses curved metal frames, which form skeleton supports for two crystal dishes; one of which is for weights, and the other for the substance to be weighed. A metal ring, of diameter equal to the distance between the middle points of the dishes when *in situ*, can be rotated by a worm-gearing, in such a way as to carry the dishes from one position to the other. This ring, which is concentric with the middle support of the balance, runs upon small rollers on the floor of the balance-case; it can be raised or lowered by an eccentric, which allows the height of the rollers to be varied; and thus the ring can be brought into action to release the crystal dishes. Sketches are given showing the shape of the curved metal dish-carriers, and the attachments to the ring which pick up the dishes. This apparatus is for use during the latter part of the weighing, that is, while the final adjustments are being made with a rider; the mechanism is arranged so that, at this point, all the necessary operations are performed from outside the balance-case. R. A.

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348. *Lunar Atmospheric Waves.* **Bouquet de la Grye.** (Ann. Bur. Long. 1895.)—The graphical representation of the variations of atmospheric pressure as observed at stations close to the sea, and therefore free from many disturbing influences, brings

out an unmistakable semi-diurnal oscillation. Curves are given for Cape Horn, Brest, St. Helena, Batavia, and Singapore. At Cape Horn the amplitude of this oscillation amounts to 3·92 mm. of water. At Brest it is only 1·3 mm., but it reaches 5·8 mm. when the perigee coincides with maximum southern declination. There is a striking analogy between the tidal wave with an equatorial amplitude of 1 m. in an ocean with an average depth of 5000 m., and the atmospheric wave of 2 mm. amplitude in a layer of air whose weight is represented by 10,000 m. of water. There is only a very slight retardation of this wave, probably owing to the extreme mobility of the upper layers of the atmosphere. At Cherbourg the atmospheric tide arrives at the same time as at Brest, whereas the ocean tide arrives four hours later. A semi-monthly atmospheric wave may also be traced in the records of twenty years at Brest. Its amplitude is 25 mm. of water. The maximum and mininum takes place two days after last quarter and full moon respectively in the latter half of the lunar month.

E. E. F.

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349. *Penetration of Solids and Semifluids by Projectiles.* **H. Resal.** (C. R. 120. pp. 397–401, 1895.)—Let the projectile be a prism moving parallel to its axis.

Let V_0 be its velocity before penetration.

V „ „ at any time t after penetration.

Ω the area, σ the perimeter } of a section of the prism.

P the weight, or $\frac{P}{g}$ the mass of the projectile.

Δ the weight, or $\frac{\Delta}{g}$ the mass of unit volume of the medium penetrated.

γ the cohesion of the semifluid or resistance to shear (*cisaillement*) per unit of surface.

N the variable reaction of the medium on the projectile.

s the path traversed through the medium.

Following the method indicated by Newton, we have

$$N dt = \gamma\sigma V dt + \frac{\Delta}{g}\Omega V dt V,$$

or

$$N = \gamma\sigma V\left(1+\frac{\Delta\Omega V}{g\gamma\sigma}\right).$$

But our author says we must write

$$N = \gamma\sigma V\left(1+\epsilon\frac{\Delta\Omega V}{g\gamma\sigma}\right),$$

where ϵ is a positive quantity less than unity to be determined by experiment.

By conservation of energy,

$$\frac{P}{g}VdV=-Nds=-\gamma\sigma V\left(1+\epsilon\frac{\Delta\Omega V}{g\gamma\sigma}\right)ds;$$

whence

$$s=\frac{P}{\epsilon\Delta\Omega}\left\{\log\left(1+\epsilon\frac{\Delta\Omega V_0}{g\gamma\sigma}\right)-\log\left(1+\epsilon\frac{\Delta\Omega V}{g\gamma\sigma}\right).\right.$$

If l be the total depth of penetration, *i. e.* the value of s when $V=0$,

$$l=\frac{P}{\epsilon\Delta\Omega}\log\left(1+\epsilon\frac{\Delta\Omega V_0}{g\gamma\sigma}\right).$$

If the projectile be a right cylinder the diameter of whose section is a, $\Omega=\frac{\pi a^2}{4}$, and

$$l=\frac{4P}{\epsilon\Delta\pi a^2}\log\left(1+\epsilon\frac{\Delta a V_0}{4g\gamma}\right).$$

It is assumed that this result holds for oblong or spherical projectiles subject to determination of ϵ.

For one and the same projectile and the same medium, the expression assumes the form

$$l=m\log(l+uV_0).$$

Let the values of l, l' corresponding to values of V_0 and V_0' where $V_0>V_0'$, be given by experiment. Let $k=\frac{l}{l'}$, then we have to determine u,

$$\log(1+uV_0)-k\log(1+uV_0')=0.$$

It is observed that $V_0>kV_0'$, whence

$$\frac{d}{dn}\left(\log+(uV_0)-k\log(1+uV_0')\right)$$

is positive if $u=0$, and the expression is maximum when

$$u=\frac{V_0-kV_0'}{V_0V_0'(k-1)}.$$

The author then compares the calculated results with those obtained by certain experiments made at Metz in 1834–35.

The experiments were made with projectiles weighing respectively 24 and 12 kg., of which M. Resal considers the former to be the more trustworthy.

He points out certain causes, *e. g.*, the hygrometric state of the earth traversed, to which may be attributed certain anomalies in the results obtained.

Using the results obtained with the projectile of 24 kg., he obtains

$$n = \epsilon \frac{\Delta a}{4g\gamma} = 0{\cdot}0048\,;$$

whence $\epsilon\Delta = 229{\cdot}007,$

$\gamma = 160{\cdot}08$ kg.

Assuming $\Delta = 1600$ kg., we obtain

$\epsilon = 0{\cdot}143.$

In general M. Resal concludes that the formula

$$l = m \log (1+uV_0) \quad \text{or} \quad l = A \log_{10} (1+uV_0)$$

may be used with safety—and it is simpler than that of Poncelet.

S. H. B.

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350. *Prytz' Hatchet Planimeter.* **E. Hammer.** (Zschr. f. Instrumk. 15. pp. 90–97, 1895.)—Description of this instrument (see 'Engineering,' May 25, 1894, p. 687; June 29, 1894, p. 813; and Aug. 24, 1894, p. 253) and tests of its accuracy, the error in which was less than 1 per cent. Herr Hammer points out in reference to polar planimeters that there is no such advantage as is usually thought in ensuring that the polar point shall remain outside the curve; and he concludes with a series of practical rules for the more accurate use of these instruments. A. D.

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351. *Forces between solid Particles totally Immersed in Liquids.* Part I. **W. J. A. Bliss.** (Phys. Rev. 2. pp. 241–258, 1895.)—If acids or salts are added to water in which finely divided clay or sand is suspended, the particles gather into groups—they become flocculated. If a trace of ammonia, caustic alkali or alkaline carbonate is added, the particles seem to become even smaller, but, as the amount of these reagents increases, flocculation is again produced. In order to investigate these effects, the author has examined the change in the distance between a fragment of very thin glass and a glass lens, when the water between them was replaced by dilute solutions. The method used was to observe the appearance of the system of Newton's rings formed by the thin layer of liquid—a contraction of the rings indicating an increase, and a broadening indicating a decrease in thickness. At first drops of a fairly strong solution were allowed to fall into the experiment vessel, but afterwards it was found better to siphon in a very dilute solution from below, and allow it gradually to displace the water which flowed out at the top. Very dilute KOH (about ·0007 gr. per c.c.) increased the separation between the glass surfaces by about 7×10^{-5} cm.; a stronger solution caused them to again approach each other. This corresponds to the flocculation

phenomena. Similar experiments with NaCl and HCl did not cause the space to become thinner, perhaps because, owing to the weight of the glass fragment, the surfaces were already in contact. Sources of error, such as the change produced by the alteration in the reflective index of the liquid, and the excess of osmotic pressure in the stronger solution, are considered, and shown not to affect the results. W. C. D. W.

352. *Forces between Solid Particles totally immersed in Liquids.* Part II. Theoretical. **W. J. A. Bliss.** (Phys. Rev. 2. pp. 373–386, 1895.)—The phenomena described in Part I are referred to the molecular forces between the solid and the liquid. By a geometrical investigation the author endeavours to replace the obvious energy conditions, which must hold in order that flocculation may occur, by an expression for the forces acting. He shows that the force, which tends to hold two submerged particles apart, may be considered to be proportional to the difference between the coefficients of molecular force between a unit volume of solid and a unit volume of liquid on the one hand, and two unit volumes of liquid on the other. He then expresses these coefficients in terms of the densities of the solid, liquid, and salt in solution, and the constants of attraction between unit masses of these substances. A rough estimate is made of the relative magnitudes which it is necessary to assign to these last constants, in order to explain the phenomena of flocculation. Thus the force between clay and water is greater than $\frac{3}{4}$ the attraction of water for itself. The attraction of ammonia for clay exceeds that of ammonia for water. KOH, and the substances of which it is typical, must have an attraction for clay greater than $\frac{3}{4}$ of that between the salt and water, while flocculating substances, such as NaCl, must have an attraction for clay less than $\frac{3}{4}$ of that for water. W. C. D. W.

353. *Liberation of Carbonic Acid from Isolated Muscles.* **J. Tissot.** (C. R. 120. pp. 641–643, 1895.)—The carbonic acid given off by an isolated muscle arises from two sources. There occur a purely physical liberation of preformed acid, and a physiological process. The quantity of CO_2 given off by a dead frog-muscle, bared of its skin, killed by immersion for 15 minutes in hot water, depends upon the temperature to which this water was raised. When the water had been heated to 50°, 0·33 c.c. were collected, when to 90°, only 0·096 c.c. of CO_2. When directly heated over mercury in a mixture of equal volumes of air and nitrogen, to 17, 36, 50, 70°, there were found, after 90 minutes, 0·15, 0·31, 0·567, 0·694 c.c. of CO_2. Of two muscles, the one tetanised, the other normal, the normal absorbed more oxygen, 0·221 against 0·200 c.c., but produced less CO_2, 0·255 against 0·424 c.c. It appears, therefore, that the CO_2 accumulates in the tetanised muscle, and is afterwards liberated as it would be from a

heated solution. Muscles kept in air for 90 minutes produced more carbonic acid than when kept in hydrogen, the numbers being 0·302, 0·208. Part of the CO_2 is therefore due to a vital process akin to respiration, consisting in absorption of oxygen and production of carbonic acid. H. B.

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354. *Approximate Frequency of Earthquake Shocks.* **F. de Montessus de Ballore.** (C. R. 120. pp. 577–579, 1895.)—The author estimates the annual total of shocks affecting a particular district from historical data i_1, from seismological observations concerning certain well-studied periods i_2, and from automatic seismographic records i_3. He finds the following ratios for 48, 28, 22 districts :—$i_2 : i_1 = 4{\cdot}26$, $i_3 : i_1 = 26{\cdot}59$, $i_3 : i_2 = 6{\cdot}44$; the last two would yield $i_2 : i_1 = 4{\cdot}18$, which is nearly equal to 4·26. Records of the first two classes would not appear to inform us of more than 3·76 and 15·54 per cent. of the actual disturbances. Adopting the intensity scale of Rossi-Forel, who distinguish ten degrees in 10,500 earthquakes observed in Italy, the author concludes that historical records generally refer only to intensities X, IX, VIII, and sometimes to VII, and that seismological data seem confined to intensities down to V. For better studied districts of 11,691,000 sq. km. area, he computes $\Sigma i_1 = 341{\cdot}35$, $\Sigma i_2 = 878{\cdot}57$, $\Sigma i_3 = 2222{\cdot}24$, and applying the above coefficients 26·59 and 6·44, $\Sigma i_3 = 16957$; that is, one shock about every half hour. Since areas of submarine disturbances are by no means unknown, we should obtain 437,768 shocks for the whole surface of the earth, about one disturbance every minute, a number which, in his opinion, nobody would have expected. No mention is made of Japan or any particular district. H. B.

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355. *The Total Eclipse of the Moon of March* 11, 1895. **J. Janssen.** (C. R. 120. pp. 524–526, 1895.)—Extensive preparations had been made at Meudon for photographic and spectroscopic observations, but during the totality of the eclipse the moon was visible at rare and short intervals only. The disc appeared less reddish than on former occasions, probably because the moon was traversing a part of the shade cone, into which few refracted rays of our atmosphere could penetrate. If the position of the moon in the cone is known, and we determine by means of photographical photometry the quantity of light reflected by the moon, we can study the refractive absorptive effects of certain higher regions of the terrestrial atmosphere. Total eclipses of the moon should further enable us to inquire into the absorption bands of oxygen in the solar spectrum, visible at sunset and sunrise, which have clearly been traced to the oxygen. The sun-light which enters the shade cone traverses a stratum of terrestrial atmosphere twice as thick as at sunrise and sunset, and should therefore show these bands, though the light would be very feeble. H. B.

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356. *Expansion Air-Pump.* **A. Raps.** (Zschr. Instrumk. 15. pp. 146–150, 1895.)—The author has modified, for laboratory purposes, the type of air-pump used for exhausting the bulbs of incandescence-lamps in which the cistern rests on a pivoted beam which by its play automatically closes the cocks. As a quick rise of the mercury might endanger the narrower glass tubes, two air-chambers have been added—the one between the starting-cock and the cock establishing connection with the cistern, the other on the other side of the first cock and in the branch leading to the water-pump. The modification necessitates the addition of a third cock, but the pump contains no valves at all. The connections are rubber pipes with internal metal coils. The sizes of the air-chambers have to be suited to the other parts. The starting-cock is provided with a screw-spindle by means of which the access of air is easily regulated. With a receiver of 600 c.c., one stroke of the pump requires from 15 to 20 seconds; a vessel of 4 litres could be exhausted to 0·001 mm. in half an hour. A vacuum of 0·000002 mm. is claimed. H. B.

357. *Density of Water at 4° C.* **J. Macé de Lépinay.** (C. R. 120. pp. 770–773, 1895.)—The author expects to obtain a value for the weight of a litre of water at 4° C. within an error of 1-1600th per cent. The measurement involves the study of the geometrical form and dimensions of a solid with reference to the standard metre, and the direct or indirect measurement of the loss of weight of this solid in pure water, free from air, at its maximum density, with reference to the standard kilogramme. The most convenient shape for the solid is that of a rectangular parallelepiped. It consists of quartz, a material whose coefficient of expansion is very well known. It is approximately a cube of 4 cm. side. The truth of the surfaces is perfect to within a few microns, and their parallelism to within 10″. The dimensions are obtained in terms of wave-lengths of light by means of Talbot's bands. The index of refraction is obtained from a piece of quartz from the same block. Absolute measurements have not yet been completed, but the experiments have shown that with a grating of 7 m. radius it will be possible to determine the thicknesses of quartz plates within an error of one-millionth. The largest error will be that attending the determination of the refractive index, but it will not exceed 6 mmgr. in the result for the weight. E. E. F.

358. *Microscope for Crystallography.* **C. Klein.** (Sitz. Akad. Wiss. Berlin, pp. 91–107, 1895.)—The investigation of crystals by the method of Biot (Mémoire sur la polarisation lamellaire, 1841, pp. 586 and 683) depends upon the immersion of the specimens in liquid media, and requires that the crystals may be rotated in either, or all, of three planes, at right angles to one another, while being examined by polarised light. In the present paper, Mr. C. Klein describes an improved apparatus which enables this method

to be applied with much accuracy. It consists of two parts :—(1) A microscope provided with polariser and analyser, and (2) a rotation mechanism for the crystal. The microscope is provided with a gearing which enables the nicol of the polariser to be turned simultaneously with the analyser, as in some English instruments. The rotation mechanism of the crystal holder is a universal joint, consisting of two brass graduated quadrants set at right-angles to one another, each moveable in its own plane about its own centre of curvature, and moveable with respect to one another in these planes The lower quadrant carries a radial arm holding the crystal at the centre of curvature. The upper quadrant is supported by a rod passing through the centre of a graduated circle ; by the rotation of this rod the crystal is turned about a vertical axis. The method of experimenting is described at some length, and numerical examples are given. R. A.

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359. *Movements of Falling Animals.* **Marey** (C. R. 119. pp. 714–717, 1894); **Guyou** (*Ibid.* pp. 717–718) ; **M. Levy** (*Ibid.* pp. 718–721); **L. Lecornu** (*Ibid.* pp. 899 900).—M. Marey, by his chronophotographic apparatus, took 60 photographs per second of the fall of a cat dropped back downwards. These photographs were then examined at the rate of 10 per second in the zoetrope, and the order of events ascertained. The order is :—(1) Reduction of the moment of inertia of the fore-quarters by packing the fore-paws close to the head ; (2) increase of the moment of inertia of the hind-quarters by spreading out the hind-legs ; (3) a twist of the vertebral column by the vertebral muscles—the consequence of which is that the fore-quarters rotate more than the comparatively unwieldy hind-quarters ; (4) spreading the fore-paws apart so as to increase the moment of inertia of the fore-quarters ; (5) bringing the hind-legs somewhat nearer to one another. The cat therefore brings its fore-quarters first into the required direction, and then the hind-quarters follow it up. M. Guyou explains this on the basis of the successive relative moments of inertia, and the angular velocities, inversely proportional to the moments of inertia, of the two parts of the body. M. Maurice Levy treats the subject mathematically, and M. Lecornu extends the mathematical theory to serpent-rotation, involving the transmission of a fibrillary contraction without any apparent change of shape.

E. Fouché (*Ibid.* 120. pp. 608–609, 1895) describes a model to illustrate the fall of animals (cats, etc.), as elucidated by M. Marey's photographs and M. Guyou's interpretation of these. A. D.

LIGHT.

360. *The Kerr Magneto-Optic Effect.* **C. H. Wind.** (V. phys. Ges. Berlin, 13. pp. 84–96, 1895.)—This paper reviews the additions to our knowledge, experimental and theoretical, of the above and kindred phenomena recently made by Dutch and other physicists. The attempt at a theory of the Kerr magnetic-effect, begun by Lorentz and extended by Van Loghem, is given first. This explanation is based upon the Hall phenomenon. Lorentz assumes that the Hall transverse electromotive intensity may be produced by a displacement current as well as by an ordinary conduction current. He then shows that the following equations may express the relations which obtain between the components of the current and the electric intensity in a magnetised medium :—

$$X_t = X_o - hv, \quad Y_t = Y_o + hu, \quad Z_t = Z_o, \quad . \quad . \quad . \quad (1)$$

where X_t, Y_t, Z_t denote the total electrical intensities, X_o, Y_o, Z_o those which arise from electrostatic and inductive actions, u and v are the currents perpendicular to the axis of z, and h is a constant which depends only upon the nature and magnetic state of the medium. On seeking what form of electrical vibrations satisfies both the above equations and those of Helmholtz, and then applying the special conditions which obtain at the interface of two dissimilar media, the following results were deduced. In the case of light reflected from a metallic mirror of either equatorial or polar magnetisation, an incident ray polarised either in or perpendicular to the plane of incidence must be resolved on reflection into two different components, one polarised in the same plane and the other polarised perpendicular to this plane. The last component will be called the *magneto-optical* component, and when its amplitude and phase are spoken of they are to be taken with respect to those of the other component of the reflected light as the standard. The theory finally arrives at expressions for the amplitude and phase of the magneto-optical component as functions of the angle of incidence, and which are further dependent only upon the ordinary optical constant of the metal, and, so far as the amplitude is concerned, on the value of the constant h. One is therefore in a position, from ordinary optical experiments and from experiments on the Hall effect giving the value of h, to deduce for any desired angle of incidence the magnitude and phase of the corresponding magneto-optical component. This theoretical value may then be compared with direct observations of the Kerr effect. Kaz' observations with iron of polar magnetisation were in fair general agreement with the above theory. Experiments with equatorially magnetised iron and extending over three years were next conducted by Sissingh. And, for each angle of incidence, on calculating from the observed rotation of the plane of polarisation the magnitude and phase of the magneto-optical component and comparing it with those derived from Lorentz' theory,

the following noteworthy discovery was made. The observed amplitudes agreed well with the theoretical, but the observed phases at each angle of incidence deviated by an almost constant value of about 85° from their theoretical values. This deviation is thenceforward referred to as the *Sissingh phase-difference* and denoted by the symbol S. Correspondingly elaborate experiments for the case of iron with polar magnetisation were then undertaken by Zeeman. These showed the existence of a Sissingh phase-difference, S=about 80°. Further researches of Zeeman clearly showed that a Sissingh phase-difference exists for cobalt also, its value being about 50°. Zeeman afterwards made a shorter set of observations with nickel, using only a single angle of incidence, and obtained the provisional value for the phase-difference, S=about 30°. It was next desirable to devise a theory of the Kerr phenomenon in harmony with these new experimental results. This was done by Goldhammer, who, starting from Maxwell's Electromagnetic theory of Light but completing the differential equations by the addition of other terms, obtained expressions for the phase and amplitude of the magneto-optical component which agreed with the final expressions of Lorentz except for a constant phase-difference. This theory, which naturally agrees quite well with the observations of Sissingh and Zeeman, is in no way based upon any immediate relation between the Hall and Kerr effects. The additional terms referred to contain an undefined complex constant. There were, therefore, two quantities to be experimentally ascertained. Drude framed a theory of this phenomenon by the addition of certain terms containing only one real constant. This led to the result that the values of S for cobalt and nickel should be about the same and of the order 60°, whereas Zeeman's experiments gave about 50° and 30° respectively. Moreover, this theory gave a dispersion which, in the case of cobalt at least, is of the opposite sense to that actually observed. Experiments of the requisite extent and accuracy with nickel were next carried out by Wind, who found for sodium light S_D=36° 44′ $\pm$ 20′·5. Zeeman has also made further experiments on nickel with different angles of incidence and finds S_D=36° 21′ $\pm$ 15′.—At this stage we have, therefore, two chief theories before us, viz.:—(1) That of Lorentz, endeavouring to connect the Hall and Kerr effects but unable to explain all the experimental facts of the latter; and (2) that of Goldhammer, more completely in accord with the observations but consisting essentially of a purely mathematical hypothesis. The author being more attracted to the former, on account of its physical basis, attempted to so amplify it as to harmonise with the known facts of the phenomenon. This he did as follows. Whereas Lorentz had assumed that the Hall effect might be produced by displacement currents, and of the *same* magnitude as when produced by ordinary conduction currents, Wind now assumes that one might, with even greater justice, suppose the Hall effect, though producible by either kind of current, to be of *unequal* magnitudes in

the two cases. Into the set of equations (1) he then introduces, for the Hall effect produced by displacement currents, a second constant k, which in general differs from h. He thus obtains

$$X_t = X_o - hv_1 - kv_2, \quad Y_t = Y_o + hu_1 + ku_2, \quad Z_t = Z_o, \quad . \quad . \quad (2)$$

where the index 1 denotes the components of the conduction current, and the index 2 those of the displacement current. In this manner expressions are obtained for the amplitude and phase of the magneto-optical component which only differ from those of Lorentz by a constant angle and which agree precisely with those of Goldhammer. At the same time one obtains a relation between S and k/h, from which the latter can be calculated. It thus appears from the determinations of S that the values for k/h are about 4·0 for nickel, a fairly large positive quantity for cobalt, and for iron a negative value of the same order as that for nickel. Now Thomson's values of h for nickel, cobalt, and iron are in the proportion $-14{\cdot}7$, $+2{\cdot}5$, $+7{\cdot}8$. If under certain conditions the value of h for cobalt might change sign, it is possible that k for all three metals might be of the same sign and of the same order of magnitude. E. H. B.

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361. *Refraction and Dispersion of Salts in Solution by an Interference Refractometer.* **A. H. Borgesius.** (Wied. Ann. 54. pp. 221–243, 1895.)—The solution under examination is contained in two small troughs having glass bottoms and floating glass tops, all with parallel plane faces. These troughs are placed side by side and immersed in a quantity of the pure solvent which is in a larger trough with plane glass bottom. The levels of the solution in the two small troughs are maintained equal by an inverted U-tube filled with the solution and whose ends dip one into each trough. Two beams of light from the same luminous source are reflected at angles of 45° from the front and back faces respectively of a partly-silvered thick plane glass plate, and pass vertically upwards through the solution, one beam through each small trough. A second reflection, under conditions precisely similar to those of the first, brings the two beams together again so as to exhibit the interference-bands whose displacement is to be observed. By means of screws, geared with toothed wheels, one of the small troughs can be raised through a given distance, the other being simultaneously lowered by the same amount. The relative motion of the two troughs is measured by a scale fixed to one of them and read by a vernier fixed to the other. Now when this raising and lowering is effected, no alteration occurs in the thickness of the layers of *solvent* traversed by each beam, only the thicknesses of the layers of *dissolved salt* are thereby changed. Moreover, since the vertical sliding motions are made gradually and without disturbing the optical steadiness of the apparatus, the number of interference-fringes which in consequence pass over the cross wire of the eye-piece may be readily counted. From this

number and the corresponding readings of the scale and vernier, the excess of the refractive index of the solution over that of the solvent alone (for the particular light used) is calculated. The solutions examined are aqueous solutions of the chlorides, bromides, and nitrates of potassium, sodium, lithium, barium, and strontium, also the sulphates of potassium, sodium, and lithium. The values of the constants of molecular refraction and dispersion are given for various degrees of concentration and at different temperatures, the tables extending over several pages. A striking regularity is noticeable in the changes of the molecular refraction constants for the changes from bromide to chloride, chloride to nitrate, &c., with the different bases. Thus, *e.g.*, with the change in the acid radicle Br—Cl, the consequent changes in the refraction constants for the bases K, Na, $\frac{1}{2}$Ba, and $\frac{1}{2}$Sr are respectively 39·1, 38·1, 39·2, and 39·1. A similar remark holds good respecting the changes in the refraction constants for a given interchange of bases with the different acids. The same regularity occurs in the changes in the dispersion constants consequent upon interchange of acids or bases. The relation between the refractive index of a solution and its concentration is discussed. In the case of sodium chloride this may be written $n=a_0+a_1c+a_2c^2$, where n is the refractive index, c is the concentration in grammes of salt per kilogramme of solution, and a_0, a_1, and a_2 are constants. The refractive index of potassium chloride, on the other hand, exhibits a maximum value for $c=4{\cdot}65$. E. H. B.

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362. *Influence of Temperature on the Transparency of Solutions.* **E. L. Nichols** and **M. C. Spencer.** (Phys. Rev. 2. pp. 344–360, 1895.)—By means of a photometer the absorption of definite parts of the spectrum by solutions of a number of substances is measured at different temperatures. The solutions used fall into 4 classes :—

(A) Solutions suffering temporary change of transmitting-power throughout the spectrum when heated : Cobalt-ammonium sulphate and potassium bichromate.

(B) Solutions suffering temporary decrease of transparency in certain regions, but not throughout the entire spectrum : Potassium chromate, sodium bichromate, copper sulphate, chromic sulphate, potassium ferricyanide.

(C) Liquids unaffected as regards transparency by temperature up to 80° : Water, cochineal, sodium nitrate, nickel sulphate.

(D) Solutions suffering permanent change of colour as the result of heating : Copper acetate, ferric chloride, potassium permanganate, copper chloride, nickel chloride.

The paper consists mainly of tables of numerical results.

J. W. C.

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363. *Photographic Photometry.* **C. E. Guillaume.** (L'Ind. Elec. No. 77. pp. 93–94; March 10, 1895.)—This is a criticism of M. Janssen's paper with the same title, an abstract of which was given on p. 163 of this Journal (April 1895). M. Janssen assumes that the density of the deposit on a developed photographic film is directly proportional to the product of the intensity of radiation which produced it and the time of exposure; so that the intensities of sources are inversely as the times they take to produce deposits of equal density. The author quotes and describes experiments of Abney (Proc. Roy. Soc. vol. 54. p. 143), in which two similar films were exposed to the same source of light for the same period, the one continuously, the other intermittently; the result on development being that the intermittently exposed film only gave 60 to 90 per cent. of the density produced by continuous exposure. Further experiments, in which different parts of the same film were exposed for different times to an invariable source of light, showed that the rate of increase of opacity became smaller as the time of exposure was increased. J. L. H.

364. *On Glow-Lamps.* **H. Armagnat.** (L'Ind. Elec. No. 77. pp. 94–96; March 10, 1895.)—The luminosity of a glow-lamp is the product of two factors—the surface of the filament and its intrinsic brightness; but the ratio of these factors determines the quality of the lamp. By increasing the brightness additional luminosity is obtained, but the increased temperature of the filament reduces the length of its life. It is therefore important to determine the intrinsic brightness of a filament under different circumstances, so that the most economical way of using it may be found. The author suggests various methods for doing this. The degree of incandescence may be determined, by comparing the ratio of intensities of radiations of two different specified wave-lengths. This ratio varies with temperature, but possibly not according to the same law for all filaments. Another way is to measure the watts per candle-power: this is a function of the temperature also, since two filaments which require the same number of watts per candle-power emit equal amounts of energy from unit surface. If this energy is all in the form of radiation the two filaments must be at the same temperature, but the convection through the rarefied gas round the filament may be considerable. A third method serves to measure directly the brightness of the filament: this is by the use of Le Chatelier's optical pyrometer, in which the intensities of the red rays from the filament and a standard amyl-acetate lamp are compared. Le Chatelier has established an empirical formula connecting the brightness of heated black bodies, such as carbon or oxide of iron, with their temperature. This method is a very simple one, but it could only be applied to new lamps, because the deposit on the globe of an old lamp would absorb an unknown fraction of the red rays of the lamp. J. L. H.

365. *Measurement of Mean Spherical Luminous Intensity.* **A. Blondel.** (C. R. 120. pp. 550–554, 1895; reference to C. R. Feb. 11, 1894.)—A metal globe, blackened within; two apertures cut out by four meridian lines, two of these opposite to one another, and the others at 18° (on the same side) to these; the source of light centred in the globe; an ellipsoidal mirror with the source of light at one focus to converge the rays, emergent from the globe, upon a ground-glass screen at the other focus. This ground-glass image serves as one of the terms of comparison in a photometer. This arrangement utilises a reflecting annulus only of the mirror, so that the coefficients of reflection of the different points of the reflecting surface do not differ very much through differences in the angle of incidence. Ten measurements are required for the whole 360°; but it is generally sufficient to take two measurements at right angles to one another, or, if the source can be made to undergo a rapid rotation, one measurement will give the mean result. Apparatus described for measuring the intensity of an arc-lamp. A. D.

366. *Application of Doppler's Principle to Radiation.* **C. E. Guillaume.** (J. de Phys. 4. pp. 24–29, Jan. 1895.)—M. Guillaume applies Doppler's principle to determine how the radiation from a moving body is influenced by the motion. If v be the velocity of wave propagation, λ the wave-length when the source is at rest, and λ' the wave-length when it approaches the observer with a velocity V, then

$$\lambda' = r\lambda, \quad \text{where} \quad r = v/(v - \mathrm{V}).$$

Using M. Wladimir Michelson's formula for the radiation from a body at rest, the author finds that the radiation from a moving body may be replaced by that of a body at rest the temperature of which is higher in the ratio of 1 to r^2, but of which the emissive power is r^7 times less. T. P.

367. *Measuring Illumination.* **E. J. Houston** and **A. E. Kennelly.** (El. World, 25. pp. 309–312, 1895.)—The authors propose to use as a measure of illumination the lowest degree of illumination with which a test object, consisting of printed characters, can be read. A box $6\frac{1}{2}$ in. (165 mm.) $\times 1\frac{1}{2}$ in. (38 mm.) $\times 2\frac{1}{4}$ in. (57 mm.) contains the test object, and is provided with a focussing eyepiece, a window closed by a translucent diaphragm of porcelain or opal glass, and a sliding shutter which may be arranged to cover any desired fraction of the window. A graduated scale is provided, and an index attached to the shutter shows the illumination. To use the instrument, the window is placed in the desired position, and the shutter is gradually closed until the test object remains just legible. The scale is graduated by experiment. The unit employed is one-twentieth of a Violle platinum standard of light, at a distance of a metre. The authors say

that the results of different observers do not appear to differ by more than the limits of probable error for the apparatus. The average deviation of a single observation is said to be 10 per cent.
A. P. T.

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368. *An Objective for a Reflecting Telescope.* **C. V. Zenger.** (C. R. 120. pp. 609–611, 1895.)—A concise description of an objective which consists of two *spherical* mirrors, one concave the other convex, of the same radius of curvature; the convex and smaller mirror is placed halfway between the concave mirror and its focus. The compound objective brings parallel rays to a focus, *behind* the concave mirror, at a distance equal to half the focal length of either of the mirrors. The advantages of the arrangement are that the aberration is very small, and the focal plane is in reality a plane and possesses no curvature. A. Gs.

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369. *Network Screens for Photo Engraving.* **C. Féry.** (C. R. 120. pp. 720–723, 1895.)—The successful reproduction of half-tones by photomechanical processes, involving the use of a lined screen, has been attributed to effects of diffraction. These effects are, however, altogether negligible, and all the facts may be explained on the ordinary theory of shadows. The best effects are obtained when the screen is placed about half a millim. in front of the sensitive plate. Each opaque square of the screen casts a shadow upon it, and as the diaphragm must be regarded as a source of light of finite dimensions, a penumbra will be formed round each shadow whose size varies with the distance between plate and screen. The best results are obtained when the penumbra has a width equal to half the total width of the shadow. This gives for the best distance of the screen from the plate the value

$$e = \frac{af}{3D},$$

where a is the width of the opaque line or square, f the distance between the plate and the diaphragm, and D the aperture of the latter. In the resulting negative the intensity of shade will be expressed by the size of the constituent points, and the reproduction of the image by any of the photomechanical processes incapable of rendering ordinary half-shades may be proceeded with.
E. E. F.

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370. *Tests of a Spring-Fuess Barograph with Rolling Weight.* **K. Scheel.** (Zschr. Instrumk. 15. pp. 133 146, 1895.)—In the novel type of barograph of the Reichsanstalt, the record board slides down, its own weight actuating the clockwork, and the style travels across the board. The barometric tube, which dips into a wide cistern, is suspended from the short arm of a beam, and balanced by a counterpoise and a rolling weight, to whose carriage the style is fixed, attached to a silk ribbon. This ribbon passes

under a pulley which rests on the common, normally horizontal armature of two electromagnets; the armature is pivoted on its centre between the two magnets, and presses the pulley against either of two rollers, which by an auxiliary clockwork are constantly turned towards each other. The slightest motion of the beam closes a Rung contact, a silver edge resting on 12 platinum wires, and puts the other electromagnet in circuit, so that the carriage moves. One sheet serves for about 5 days; by using styles with coloured inks, the same sheet has occasionally done duty for a fortnight. In the older instruments, as in use at Pawlowsk, the mercury meniscus in the cistern was broken every few minutes; yet, owing probably to surface-tension, the barograph was always slightly in advance, both with a rising and a falling barometer. In the new type a wooden plunger entering from the side shakes the mercury up every five minutes; the instrument is now always a little behind, by less than 0·01 mm. The paper discusses the various sources of error at length. H. B.

371. *Ultra-Violet Rays of the Corona, Total Eclipse of April* 16, 1893. **H. Deslandres.** (C. R. 120. pp. 707-710, 1895.)—With the help of quartz and Iceland spar lenses fixed in a large polarsiderostat, Deslandres photographed the ultra-violet spectrum during the whole forty minutes the totality lasted; an iron spark-spectrum was added immediately afterwards. The radial slit cut the image about the equatorial diameter. The spectrum is continuous and contains bright, but no dark lines. By comparison with the iron spectrum, it was ascertained that the coronal spectrum extends to λ 300, whilst the disc emits rays to λ 295, but the former is faint. Its intensity is greatest in the blue, where it reaches a height of $\frac{2}{3}$ the solar diameter; towards the violet the spectrum diminishes both in height and intensity; at λ 300 it is reduced to a line. Previous observations had already established that the maximum intensity of the coronal spectrum is in the red. Forty lines are tabulated from λ 308·98 to λ 362·90; their intensities were determined by comparison with the H and K of the calcium spectrum. The ray 317·09 which extends to a great height in the solar atmosphere, and several others, seem comparable to H, Hε, Hζ of hydrogen. Most of the rays may belong to either the chromosphere or the corona, more probably the latter. They have not been identified, and appear to represent gases of low atomic weights. H. B.

HEAT.

372. *Latent Heats of Fusion and Vaporisation.* **M. Berthelot.** (Ann. de Chim. et de Phys. 4. pp. 133–136, Jan. 1895.)—The law according to which the molecular latent heat of vaporisation of homologous bodies is proportional to their absolute temperature leads to some important conclusions respecting the calculation of variations of entropy, in every double decomposition between bodies for which the law holds good, such as the normal alcohols of the series $C_nH_{n+2}O$ and the acids $C_nH_{2n}O_2$. In such a reaction, when an alcohol is replaced by its homologue, the variation of entropy due to the change of state from the liquid to the gaseous is *nil*. The same holds good if we replace an acid by its homologue or by an alcohol, or either of these by a molecule of water. The same relations probably hold good in a great number of cases of fusion. E. E. F.

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373. *Heat Spectra.* **E. Carvallo.** (Ann. de Chim. et de Phys. 4. pp. 5–79, Jan. 1895.)—The determination of wave-lengths in the invisible spectrum by the method of interference-fringes is subject to the disadvantage that the minima are not sufficiently strongly marked. The author describes two new methods based upon the fact that the intensity of the fluted spectrum changes most rapidly at the points where the quartz plate produces between its ordinary and its extraordinary ray a difference of phase of $\frac{1}{4}$ wave-length. If ϕ is the difference of phase, the intensity i of the fluted spectrum is deduced from the incident intensity I by the formula

$$i = \mathrm{I}\cos^2\pi\phi.$$

If now the polariser or the analyser are turned through 90°, so that they are crossed, the new intensity is complementary to i, and is given by

$$i' = \mathrm{I}\sin^2\pi\phi.$$

Also

$$\frac{i-i'}{i+i'} = \cos 2\pi\phi.$$

This quantity is determined for positions of the telescope varying from 2 to 2 minutes of arc. From these determinations the position of the thermopile corresponding to

$$\phi = \pm\tfrac{1}{4}+k, \quad \text{or} \quad \cos 2\pi\phi = 0,$$

is deduced. The author further investigates in detail the best construction of the birefracting analyser, the differential thermopile, and the various sources of error. He concludes by advancing some far-reaching claims as to accuracy and speed of this new method of infra-red spectrometry. The method, according to

him, equals that based upon the utilisation of Fraunhofer's lines by the most perfect instruments. It has errors ten times less than those of any other calorimetric method, and is ten times more rapid. His results confirm the necessity of Briot's dispersion term. The differential equations of light must contain an effect proportional to the elongation of the ether. E. E. F.

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374. *Thermal Properties of Vapours.*—Part 4. **Angelo Battelli.** (Ann. de Chim. et de Phys. pp. 408–431, Nov. 1894.) —This portion deals with the application of the laws of Boyle and of Gay-Lussac to steam. The conclusions arrived at are:— (1) That the vapour-tension of water, in the first moment of condensation at a given temperature, is generally smaller than its maximum tension. The ratios between the two tensions have a tendency to diminish as the temperature rises; while at low temperatures they approach unity, probably owing to insufficient sensitiveness of the apparatus. (2) The maximum vapour-tensions of water cannot be represented by a single Biot's formula between the wide limits of these experiments, but three different ones must be employed—one for temperatures between $-10°$ and 100° C.; another for 100° to 250°; and a third for 250° to 364°. (3) The coefficients of expansion at constant volume diminish with the temperature, and that more rapidly as saturation is approached. Besides, the absolute values and the variations of these coefficients increase with the pressure of the vapour. (4) The coefficient of increase of pressure at constant volume diminishes with increasing temperature, and always more rapidly when the volumes are smaller. But the absolute values of these coefficients are greater with small volumes. (5) The differences

$$a = \frac{p_1 v_1}{pv} - 1$$

(where $p_1 v_1$ belong to the state of gas and pv to the state of vapour) increase for each temperature as the vapour approaches saturation; and when taken near saturation, these differences increase as the temperature rises. (6) The ratio

$$\frac{pv}{p'v'\sqrt{\mathrm{T}}}$$

of Herwig's formula (where pv belongs to the gaseous state and $p'v'$ to that of saturated vapour) gradually diminishes till about 190° C., when it attains a minimum and begins to increase. (7) The formula of Clausius is applicable to water in the form given by the author for the case of the other vapours:

$$p = \frac{\mathrm{RT}}{v-a} - \frac{m\mathrm{T}^{-\mu} - n'\mathrm{T}^{v}}{(v+\beta)^2}.$$

The vapour-pressures of water are given at constant volume by the formula

$$p = b\mathrm{T} + a,$$

where b and a are two constants. The author has compiled a table of vapour-pressures for 1 gr. of steam, for volumes ranging from 3 c.c. to 20,000 c.c. (9) Admitting the hypothesis that double molecules are formed in water vapour, the number of these groups at the moment of condensation increases rapidly with high temperatures, and above 320° groups of three, four, and more molecules must be formed. E. E. F.

ELECTRICITY.

375. *Theory of Electrolysis.* **A. H. Bucherer.** (El. World, 25. pp. 138–140, 1895.)—The basis of the new hypothesis is the mass law of chemical action. From the standpoint of this law chemical equilibria are viewed as dynamical equilibria. If, for instance, it is found that at a certain temperature and pressure HCl, NH_3, and NH_4Cl are in equilibrium in such a manner that the partial pressure of NH_4Cl is three times as great as that of HCl and NH_3 singly, then this equilibrium is explained by saying that the velocity with which NH_3 and HCl form NH_4Cl is three times as great as that of the reverse action. These two reactions are imagined to occur continually, so that under the conditions characterising the equilibrium the mixture of the gases never varies in composition. Now it can be shown that the maximum amount of work which can be derived from such an action as $NH_3 + HCl = NH_4Cl$ depends on the partial pressures with which the three different gases participate in the equilibrium. If the gases are taken in unit concentration, namely, one gram-molecule per litre, this maximum work will be

$$W = RT \log_e \frac{P_1}{P_2 P_3},$$

where P_1, P_2, P_3 are the partial pressures of NH_4Cl, NH_3, and HCl respectively. The maximum amount of work is not dependent on the manner in which it is obtained, provided the manner is a reversible one. If the above reaction could be used in an electrochemical system, the E.M.F. derivable must be equal to the value

$$RT \log_e \frac{P_1}{P_2 P_3},$$

when one electrochemical equivalent is involved in the reaction.

S. S.

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376. *Potential Difference between Metals and Electrolytes.* **V. Rothmund.** (Zschr. phys. Chem. 15. pp. 1–32, 1894.)—In Lippmann's capillary electrometer the mercury and dilute sulphuric acid have a potential-difference corresponding to the contact of these two substances. If the + mercury is gradually charged negatively, its charge will at first fall to a minimum and afterwards increase. If s is the area of the mercury, γ its surface-tension, π the potential-difference, and ϵ the electric charge, it has been shown that $d\gamma/d\pi = -\epsilon/s$. From this equation it follows that when the mercury has no charge, the surface-tension is a maximum. This gives a method of determining the contact-difference of potential between mercury and dilute sulphuric acid, and if fluid amalgams be substituted for the mercury and various electrolytes for the sulphuric acid, it is possible to find the potential-difference

for a series of metals and electrolytes. The author gives the following results determined in this way :—

Hg	in normal	H_2SO_4	.....	+0·926 volt
Hg	„	HCl	.....	+0·560 „
Pb amalg.	„	H_2SO_4	.....	+0·008 „
Bi „	„	H_2SO_4	.....	+0·478 „
Sn „	„	HCl	.....	+0·080 „
Cu „	„	H_2SO_4	.....	+0·445 „
Cd „	„	H_2SO_4	.....	−0·079 „
Zn „	„	H_2SO_4	.....	−0·587 „
Tl „	„	HCl	.....	+0·089 „

The electrolyte in the case of the experiments with amalgams contained a trace of the sulphate or chloride of the metal in the amalgam. Cells put up with these amalgams against 'normal electrodes' gave the following results :—

	Found.		Calculated.
Pb	0·923	.....	0·918
Bi	0·437	.....	0·448
Sn	0·534	.....	0·480
Cu	0·458	.....	0·481
Cd	1·090	.....	1·005
Zn	1·472	.....	1·513
Tl	0·652	.....	0·471

Further results are given for the potential-difference between mercury and complex electrolytes. These direct measurements show that the E.M.F. of a galvanic cell is the algebraic sum of the observed potential-differences; and the potential-difference between two metals or two liquids must be zero, or at highest a few hundredths of a volt. S. S.

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377. *Potential Gradient in a Positive Glow Discharge.* **A. Herz.** (Wied. Ann. 54. pp. 244–264, 1895.)—An unstriated positive glow-discharge is produced in a vacuum by means of 1040 Planté cells. The potential-difference between two points in the discharge is then measured by a standardised quadrant-electrometer, and thus the potential gradient calculated. Tubes of various diameters are experimented with, and contain either hydrogen or nitrogen (the latter prepared from air). The degrees of exhaustion employed are of the order 1 to 10 mm. of mercury. The currents used vary from 0·25 to 2·3 milliamperes. For tubes of 15 mm. diameter containing gas at a pressure of 4 mm. of mercury, the following values were obtained. With a current of 1 milliampere in hydrogen, v, the potential gradient in volts per cm., $=64·5$. With a current 1·2 milliampere in nitrogen, *ceteris paribus*, $v=92·2$. The experimental results are embodied in fifteen tables; the chief of them may, however, be summarised as follows :—(1) The potential

gradient, v, in a positive unstriated glow-discharge with constant current decreases with increase of the current, i, and the equation

$$v = v_0 - b(i - i_0)$$

gives with sufficient accuracy the magnitude of this decrease. The quantity b—that is the decrease of the potential gradient for unit increase of the current—decreases with increasing width of the luminous discharge in the tube, is almost independent of the pressure, and has about the same value for nitrogen and hydrogen. (2) The potential gradient decreases with increasing width of the luminous discharge and increases with increasing pressure, but slower than the pressure increases. (3) At pressures between 4 and 8 mm. of mercury, the potential gradient for nitrogen free from oxygen is 1·4 times that for hydrogen. E. H. B.

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378. *Theory of Electrolytic Dissociation.* **S. Tanatar.** (Zschr. phys. Chem. 15. pp. 117–123, 1894.)—The low molecular conductivity of alcoholic solutions, as compared with aqueous solutions of corresponding concentration, might be due either to smaller dissociation, or to increased ionic friction. The inversion coefficient for cane-sugar is reduced much less than the conductivity by the addition of alcohol, which indicates that a great part of the effect is due to increased friction. The author examines the question of dissociation by experiments on the heat evolution when ethyl alcohol is mixed with aqueous solutions of hydrochloric acid. The reaction coefficient, and therefore the dissociation, of HCl decreases by 30 to 35 % when it is transferred from a solution in water to a solution containing 50 % by volume of alcohol. The dissociation of a gram-molecule of HCl in water is attended by a heat-evolution of 17·4 cal. We should expect the change of solvent, therefore, to be attended by an absorption of heat of about 3 cal.; or if, as in these experiments, a 2-normal solution is mixed with an equal volume of pure alcohol, we should look for an absorption of about 6 cal. per litre, which would reduce the heat-evolution, caused by the mixture of solvents, by that amount. The mixing of equal volumes of alcohol and water gave a heat-evolution of 15·825 cal., while, when the water was replaced by a 2-normal solution of HCl, the mixing developed 15·560 cal. Thus the difference is very small; and the author concludes that the dissociation of HCl in a 2-normal aqueous solution is nearly equal to that in a 44 % (by mass) alcoholic solution, in which the concentration of the acid is 1·03-normal. The decrease in molecular conductivity, therefore, must be due to increased ionic friction. The mere dilution, caused by mixing, would develop only a small quantity of heat; further dilution with an equal volume of the mixed solvent gave an absorption of 0·180 cal. per mol. of HCl. Equal masses of water and alcohol were then mixed—by volume 250 c.c. water and 300 c.c. alcohol. The heat-evolution was 15·044 cal. When the water was replaced by a 2-normal aqueous solution of

HCl, the evolution was 16·062 cal. Does this show greater dissociation? Two molecules of KOH passing from 2-normal aqueous to 1·03-normal 44 % alcoholic solution absorbed 1·899 cal., and to 0·938-normal 50 % alcoholic solution, 1·288 cal. The heat of neutralisation of 1·03-normal solutions of HCl and KOH in 44 % alcohol was 14·512 cal. per molecule, and of 0·938 normal solutions in 50 % alcohol, 13·599 cal. Experiments were also made with KCl and other salts. The heat absorbed on solution in mixtures of alcohol and water is in each case greater than that absorbed by solution in pure water. This cannot be due to increased dissociation, as in water it is already practically complete. The depression of the freezing-point, produced by KCl in a mixture of 1 litre of water and 200 gram. of alcohol, is greater than the depression it produces in pure water.

[The author seems to assume that the heat of dissociation is independent of the solvent. Is this justifiable? The heats of dissociation of different molecules are calculated by a thermodynamical equation from the temperature coefficients of dissociation, and would be affected by any forces between the solvent and the molecules or the dissociated ions.] W. C. D. W.

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379. *Photo-electric Currents.* **J. Elster** and **H. Geitel** (Sitzungsber. d. Berl. Akad. d. Wiss. 11. pp. 209–216, 1895.)—When a beam of light is made to fall upon the surface of a kathode in a vacuum-tube connected with a battery giving a high E.M.F., a current passes whose strength varies with the angle of incidence and with the plane of polarisation of the light. The authors use the liquid alloy of potassium and sodium in equivalent proportions, contained in a glass vessel 5 cm. in diameter, the anode being a piece of platinum wire. They operate with an E.M.F. of 420 volts, and with a beam from a zircon disc made white-hot in an oxy-coalgas flame, the light being passed through a nicol. If α is the azimuth of the polarised ray, counted from the position in which the plane of polarisation is normal to the plane of incidence, *i. e.* when the electric displacements occur in the plane of incidence, and if A and B are the intensities of the photo-electric current for $\alpha=0°$ and $\alpha=90°$ respectively, the current-intensity may be represented by the formula

$$J = A\cos^2\alpha + B\sin^2\alpha.$$

A and B depend upon the angle of incidence. Apart from zero-angle of incidence (for which A = B), A is always greater than B, and increases with the angle of incidence, until the latter at about 60° approaches the angle of polarisation for the kathode substance, when it begins to diminish. B diminishes throughout as the angle of incidence increases. At angles between 60° and 65° the ratio A : B is about 50 : 1. The preponderance of A explains the fact that ordinary light also gives a maximum current at about 60°.

There is probably some connection between the greater efficiency of light polarised at right angles to the plane of incidence and its further penetration into metallic sheets discovered by Quincke.

E. E. F.

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380. *Induction-Tests by Telephone and Bridge.* **H. Abraham.** (J. Phys. 4. pp. 127–132, March 1895.)—This is a note with regard to the employment of the telephone in conjunction with the Wheatstone bridge. The general expression for the square of the maximum value of the current through the telephone is given, and the conditions that this shall be zero are determined. If the adjustments are made at only one branch of the bridge, the sound in the telephone may be brought to a minimum, but will not necessarily be entirely extinguished; equations corresponding to this preliminary adjustment are derived from the general expression. The resistance, r, to be added to one of the branches, R_4, in order to re-establish balance when continuous currents are used, is represented by the equation

$$r = \frac{4\pi^2 L_2 L_3}{R_1} \cdot n^2,$$

in which r is seen to depend upon the square of the frequency. Experimenting with a bridge where the other values are known, the author measures r, and uses it as a check for determining n. The results are accurate to within 1 per cent. of the stroboscopic observations. The alternator could furnish 12000 ~ per second (6000 periods). It consisted of a small mouse-mill, having 40 iron bars placed parallel to one another around a common axis. This cylindrical cage was revolved before the magnet of an Ader telephone, which generated currents of sufficient strength to cause a receiving telephone connected with it to emit a sound audible at a distance of several metres. Further, by determining r, and using alternating currents of known frequency, the author finds the geometric mean of the coefficients of induction in two of the branches, in absolute measure, in terms of a *resistance* R, and a *time*, by the relation

$$\sqrt{L_2 L_3} = \frac{1}{2\pi n} \sqrt{r R}.$$

The rest of the paper has reference to the comparison of small coefficients of induction by the telephone method. The bobbins to be compared are placed in branches (1) and (2) of the bridge, the opposite branches being without induction ($L_3 = L_4 = 0$). It is then shown that, if the frequency is great, the required ratio L_2/L_1 is nearly exactly equal to R_3/R_4, corresponding to minimum sound; and this is true even when the effect of the induction is such as to necessitate a very considerable alteration in the bridge adjustment from that corresponding to a continuous-current balance. R. A.

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381. *Electrostriction by Free Ions.* **P. Drude** and **W. Nernst.** (Zschr. phys. Chem. 15. pp. 79–85, 1894.)—Dielectrics, whose

specific inductive capacities increase on compression, contract when under the influence of an electrostatic field. If free ions exist in solutions of electrolytes, it therefore follows that, under the influence of the electrostatic field which their charges produce, the solvent must be compressed. Taking ϵ to denote the specific inductive capacity, r the distance from the centre of an ion, v the volume of the solvent, k the bulk modulus, e the charge on an ion whose radius is ρ, the authors deduce, from the usual expression for the energy of an electrostatic field, a value for the change in volume produced by the ionic charge—

$$\Delta v = \tfrac{1}{2} e^2 \int_0^\infty \frac{1}{k\epsilon^2} \left(\frac{d\epsilon}{dv}\right) \frac{dr}{r^2},$$

which, since ϵ and $\frac{d\epsilon}{dv}$ only change slowly, gives

$$\Delta v = \frac{1}{2k} \cdot \frac{e^2}{\rho} \cdot \frac{1}{\epsilon^2} \frac{d\epsilon}{dv}.$$

The density of salt solutions, in all cases, shows that contraction has occurred on solution, and, as the dilution and the dissociation increase, the calculated molecular volume of the salt becomes less, indicating that the contraction of the solvent, due to a given amount of salt, has increased. The molecular volume of undissociated acetic acid is 51 c.c. The molecular volume of the same compound, when completely dissociated, is found by adding that of sodium acetate (40·0 c.c.) to that of hydrochloric acid (18·0) and subtracting that of sodium chloride (16·5). The result (41·5 c.c.) shows that the contraction produced by the dissociation is

$$51{\cdot}0 - 41{\cdot}5 = 9{\cdot}5 \text{ c.c.}$$

In a similar manner the electrostriction, produced in the same solvent, water, by the dissociation of phosphoric acid, was found to be 11·3 c.c. It is also possible to calculate it directly. Thus, in normal solutions, where the dissociation is 15 per cent., the molecular volume of phosphoric acid is 46·6 c.c., and in 0·0025 normal solutions, where the dissociation is 90 per cent., the molecular volume is 37·3 c.c. This gives for the electrostriction produced by full dissociation the value 12·4 c.c. By the same method the electrostriction of water was calculated from solutions of monochloracetic acid to be 10·6 c.c., and from solutions of tartaric acid to be 8·1 c.c. Thus the electrostriction produced by the dissociation of one gram-molecule into two monovalent ions is constant, within the limits of experimental error, for a number of cases, and has the value 8 to 11 c.c. The authors also deduce from the equations that the electrostriction of water depends on the magnitude of the charges on the ions and not on the sizes of the ions. Hence electrolytes which give divalent ions with double charges must produce greater electrostriction. W. C. D. W.

382. *Action of a Synchronous Motor.* **F. Bedell** and **H. J. Ryan.** (J. Frank. Inst. No. 831, pp. 197–214, March 1895.)—There is nothing new in the paper. Diagrams of the various phase relationships between the current and the E.M.F.'s of generator and motor are obtained by direct experiment and compared with those obtained from a consideration of general principles and *ab initio* reasoning. Some of the statements with regard to self-induction and armature reaction are open to criticism. Perhaps the most interesting part of the paper is the description of the Bedell-Moler phase-indicator and synchroniser. The diagrams emphasise the necessity of considering armature reaction when determining the working conditions of an alternating-current plant. W. G. R.

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383. *Potential of an Electrified Surface.* **J. Andrade.** (C. R. 120. pp. 605–608, 1895.)—This paper contains another proof of the formula connecting the change of induction on crossing an electrified surface with the surface-density of the electrification. An expression is found for the repulsion of a unit of electricity at a point M on the normal to the surface at M_0, and the difference in the values of the expression for points M and M′ on opposite sides of the surface is deduced. This difference approaches a limit as M and M′ approach each other, the limit being $4\pi\sigma$, where σ is the surface-density of the charge at M_0. J. L. H.

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384. *Therapeutic Action of High-Frequency Alternating Currents.* **Apostoli** and **Berlioz.** (C. R. 120. pp. 644–645, 1895.)—The authors placed 75 patients in the great solenoid of D'Arsonval, many times during the day, for 15 or 20 minutes each time, and continued this treatment from January 1894 till March 1895, so that each patient was electrified 2446 times. The results were negative in certain cases of hysteria and local neuralgia. The best results were obtained with patients suffering from arthritis, rheumatism, gout, and glucosic affections. In these cases appetite, sleep, and general strength improved even before the urine showed any marked change. Then uric acid decreased, whilst the urea increased. Three patients suffering from diabetes, and not otherwise treated, secreted considerably less sugar. H. B.

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385. *Effect of an Alternating E.M.F. on a Capillary Electrometer.* **B. Brunhes.** (C. R. 120. pp. 613–615, 1895.)—The surface-tension of the mercury in a Lippmann electrometer is maximum for an E.M.F. of 0·95 volt; there is then no double electric film at the contact mercury/dilute acid. Any additional E.M.F. applied has a certain effect which is independent of the sign of the current, and this effect should show equally in the case of alternating currents. This the author has established with the help of a commutator of the type of which Bouty availed

himself for determining electrolytic resistances. The commutator was turned by a small electromagnetic motor. When not starting, however, from the maximum capillary constant, the alternating current should not and does not affect the meniscus. Up to 2000 alternations per minute were employed. It remains to be seen, whether the results would be the same at much higher frequencies, as a certain time may be necessary for charging the electrometer.

H. B.

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386. *Accumulators with Mercury Electrodes.* **L. Poincaré.** (C. R. 120. pp. 611–613, 1895.)—Accumulators with liquid electrodes would be preferable to lead cells, inasmuch as their electrodes would be uniformly attacked and would not disintegrate. Mercury in an acid electrolyte will not answer, on account of the generation of hydrogen. In the alkali haloids, however, the mercury cathode becomes amalgamated, and we obtain a reversible cell of about 2 volts, with the only drawback that chlorine and bromine form poorly conducting layers on the anode. There remains sodium iodide which, provided neither the current density nor the temperature be too high, yields a very good accumulator of 1·85 volts. The solution of the iodide of sodium must be concentrated so that all the iodide of mercury forming may dissolve. The two concentric mercury electrodes are separated by a thin glass cylinder, reaching almost to the level of the mercury. There is very little loss of sodium; the efficiency is about 90 per cent., and the capacity that of ordinary accumulators. The cell may be discharged at any rate without detriment. Although the arrangement will hardly lend itself to adoption in practice, it is theoretically interesting that under certain circumstances the E.M.F. is independent of the temperature, so that then the chemical energy is wholly recovered as electrical energy. H. B.

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387. *A Thermo-Chemical Carbon Pile.* **D. Korda.** (C. R. 120. pp. 615–618, 1895.)—When metallic salts are reduced by carbon, heated to dark red heat, electric currents are generated, as Korda has been able to demonstrate in two cases. Barium dioxide becomes positive to carbon, to the extent of about one volt. Other dioxides—Cu, Mn, Pb—do not show any free electricity, because the reduction to metal gives rise to local currents. When the copper dioxide is separated from the hot carbon by a layer of carbonate of potassium, however, a current of 1·1 volts is observed. With the barium compound, the reaction takes place according to the formula

$$2BaO_2 + C = CO_2 + 2BaO.$$

Since $C,O_2 = 97{\cdot}65$ cal., and $BaO,O = 12{\cdot}1$,

$$97{\cdot}65 - 2 \times 12{\cdot}1 = 73{\cdot}45 \text{ cal.}$$

should become evolved, corresponding to 1·58 volts. The experiments were made by placing a lump of BaO_2 of 2 or 3 c.c. on a plate

of carbon, connecting both by a platinum wire with a Richard voltmeter, and heating the carbon directly by means of a Bunsen burner, or indirectly in a crucible. The E.M.F. rose quickly; in the crucible the internal resistance was 13·6 ohms. The copper dioxide was heated in a crucible; the E.M.F. was

	0·1,	0·9,	1·0,	1·1,	1·0,	0·9 volts
after	9,	15,	34,	45,	61,	75 minutes;

the resistance 3·2 ohms. When the potash was damp, the current first flowed from the carbon to the copper, to reverse afterwards.

H. B.

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388. *Electrical Conduction of Rarefied Air.* **C. A. Mebius.** (Wied. Ann. 54. pp. 520–543, 1895.)—In a tube half a metre long are two flat electrodes, one of which is moveable. The tube is in circuit with a battery of storage-cells, a galvanometer, a telephone, and a large adjustable resistance. The difference of potential between the electrodes is measured by joining a branch circuit to them containing a galvanometer and a resistance of 40 million ohms. Observations are made of the difference of potential between the electrode with different distances between them and different current strengths, the tube containing air at pressures of $\frac{1}{4}$ to $\frac{3}{4}$ mm. The result for any one distance between the electrodes can be expressed by an equation of the form $V=a+bi$, where b is generally smaller and a greater, the greater the length of the discharge. After the current has passed for some time the potential fall increases, and simultaneously the negative glow expands and the positive column contracts, until at last the current becomes discontinuous. The author attributes the change to the formation of oxides of nitrogen.

J. W. C.

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389. *Resistance of Solutions.* **F. Kohlrausch** and **A. Heydweiller.** (Wied. Ann. 54. pp. 385–395, 1895.)—The resistance of aqueous solutions and of distilled water decreases when a current passes. This decrease, with an E.M.F. of 200 volts, begins a few seconds after the current is made. After 2 or 3 minutes the resistances rise again. On reversing the current the resistance rises abruptly or very rapidly, and then begins to fall. In the case of water which had stood in the glass vessel for 18 hours, the decrease amounted only to 20 per cent. under circumstances where purer water showed a decrease of nearly 50 per cent. But the rise on reversal amounted to five times the original value. A 0·01 normal solution of K_2SO_4 showed a fall of about 60 per cent. in 67 seconds, with an E.M.F. of 200 volts. After 20 minutes it had risen to 50 times its original resistance. Mi ing the liquid restored the initial value. A 0·001 normal solution of NaCl fell in 6 minutes to $\frac{1}{3}$ of the original resistance, but did not rise after that. HCl and KOH solutions showed an

increase of resistance, which was partly counteracted by reversing the current. The decrease observed in saline solutions is accounted for by supposing that H and OH proceed from the anode and the kathode, and thus increase the conductivity. Arrived in the middle, they combine to water, and a layer of pure water is formed in the centre. If this could be done on a large scale, it would be a convenient method for producing pure water. But in larger vessels the process is interfered with by convection currents. The formation of this layer of pure water accounts for the subsequent increase of resistance. In the case of pure water it may be assumed that more highly conducting H and O compounds are at first formed, which are decomposed by a lasting current. It is significant that the initial value of the resistance is only gradually restored by shaking, and not instantly, as in the case of saline solutions. E. E. F.

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390. *A Method for preventing Armature Reaction.* **H. J. Ryan** and **M. E. Thompson.** (T. Am. Inst. El. Eng. 12. pp. 157–181, 1895.)—The authors claim to have completely nullified armature reaction in direct-current machines by means of balancing-coils placed in series with the armature. The balancing-coils cross each pole-face, and possess a number of ampere-turns equal to and opposite in direction to the number of ampere-turns on the corresponding part of the armature. To satisfy completely the necessary conditions for perfect commutation in a constant potential dynamo, the gap between the pole-pieces is bridged across, a commutation-lug is attached to the centre of this bridge, and the lug is made the centre of the balancing-coil, which is now provided with a few extra turns. By this means the authors have attained sparkless commutation at all loads without shifting the brushes; and, in addition, by simply shifting the brushes, the compounding of the machine from 10 per cent. drop to 10 per cent. over-compound, without interfering with the commutation. By balancing the armature reaction in this way, the output of a machine is greatly increased, the upper limit of load depending only on the heating of the armature-coils. The authors do not claim originality for the idea of using balancing-coils, but only for the combination of balancing-coils and commutation-lug. The curves connecting output and volts at the brushes show the immense advantage of neutralising armature reaction.
W. G. R.

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391. *Electric Conductivity of Salt Solutions.* **A. C. MacGregory.** (Phys. Rev. 2. pp. 361–372, 1895.)—The electric conductivity of aqueous solutions of certain salts, which have not hitherto been fully examined, were measured, for different dilutions, by Kohlrausch's method. Tables are given showing the conductivity and molecular conductivity of the following salts:—Calcium sulphate, nitrate, chloride and acetate; strontium chloride,

nitrate and acetate; barium acetate; silver acetate; potassium oxalate. The molecular conductivities continually rise, as the dilution increases, in all cases but that of silver acetate. For this salt, the molecular conductivity attains a maximum at a concentration of about 0·0006 gram-equivalent per litre, and then falls again as the dilution is pushed still farther. The author says he is unable to explain this phenomenon, which has been observed for no other salt. [In some measurements I am now making, the same thing has appeared with solutions of ferric chloride. In this case it is due to the fact that the salt decomposes into free acid and soluble ferric hydroxide. This decomposition also occurs with ferric acetate. I expect the same explanation holds good in the case of silver acetate, discovered by MacGregory.]

W. C. D. W.

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392. *Specific Inductive Capacities.* **H. Pellat.** (C. R. 120. pp. 773–775, 1895.)—The apparatus consists essentially of a Kelvin double absolute electrometer. The two moveable aluminium plates are placed horizontally one above the other, and are rigidly connected. The combination is suspended by one arm of a balance, the other arm carrying a pan provided with an air-damper after P. Curie. The guard-rings are connected by a metallic cylinder, and all the parts mentioned are in metallic connection with the cage. The attracting-plates are connected to each other, but are otherwise insulated. One of them is fixed, and the other may be adjusted by a micrometer-screw. A fixed microscope provided with a cross-wire is used for observing another one fixed to the bar connecting the two moveable plates. The two cross-wires coincide when the upper moveable plate is exactly in the plane of its guard-ring. A spring of very fine silver wire is fixed to the beam of the balance, and is held up by a cocoon fibre wound round a small drum. This enables the observer to adjust the balance to the position of equilibrium. To find the sp. ind. cap. of a slab of dielectric about 0·8 cm. thick, it is introduced between the upper moveable and attracting plates, resting upon the guard-ring of the latter by three glass feet. Everything being at the same potential, the position of equilibrium is obtained. Then a difference of potential is established—it may vary within wide limits; and by suitably varying the position of the attracting plate two positions are found very close together, such that the commutator makes the balance move in opposite directions. The mean position, which corresponds to equality of attraction of the two moveable plates, is read on the head and scale of the micrometer-screw. Then the dielectric is removed and the same operation repeated with air, when the attracting-plate will have to be screwed into another position at a distance e from the first. Then the spec. ind. cap. of the material is $K=\frac{c}{c-e}k$, where c is the thickness of the dielectric, and k the spec. ind. cap. of air.

E. E. F.

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393. *The Permeability of Mild Steel.* **D. C. Jackson.** (El. World, 25. p. 332, 1895.)—The author gives the permeability curves (μ B curves) for seven samples of mild steel, which confirm the opinion previously expressed by him, that the presence of manganese in as large a percentage as 0·6 per cent. exerts a markedly injurious effect on the magnetic quality of steel-castings.

W. W.

---

394. *The Ratio between the Lateral Contraction and the Longitudinal Dilatation (Poisson's ratio) in Magnetised Iron Rods.* **A. Bock.** (Wied. Ann. 54. pp. 442–451, 1895.)—The author makes use of the form of apparatus employed by Kirchhoff (Pogg. Ann. 108. p. 369, 1859), in which the torsional rigidity and Young's modulus are determined simultaneously, and the value of Poisson's ratio is deduced from the formula $\mu = \frac{T}{B} \cdot \frac{s}{2l} - 1$, where T and B are proportional to the torsional rigidity and Young's modulus respectively, s is half-length of the rod, and l length of the lever-arm, at the end of which the weight employed to bend and distort the rod acts. In the case of hard steel, the author concludes that if any change at all takes place on magnetisation in the rigidities or in Poisson's ratio, this change must be less than 0·5 per cent. When experimenting on soft iron, the rod passes through the inside of a magnetising-coil, the heating of the rod being prevented by drawing a current of air through the annular space between the inside of the coil and the iron rod. Although the changes on magnetisation in the elastic constants are very small, not amounting to 0·5 per cent., the author considers that his experiments show that the bending and the modulus of torsion diminish, but the former to a greater extent, so that Poisson's ratio increases. The author has noticed that repeated magnetisation and deformation makes the differences observed between the elastic constants in the magnetised and unmagnetised states diminish, and hence he concludes that the attempts to detect the effect of magnetisation by the change of pitch of a rod's note must be fruitless.

W. W.

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395. *Electrolytic Disintegration.* **C. J. Reed.** (J. Frank. Inst. 832. pp. 283–286.)—The author finds that by exposing a very small surface of a negative lead electrode to a solution of sodium phosphate in an electrolytic cell, there is formed a voluminous cloud of dark blue or lead-coloured precipitate, which sinks through the solution and renders it opaque. This precipitate appears to be finely divided lead torn from the negative electrode. Other electrolytes which give this effect are generally those which yield hydrogen directly or as a secondary product of electrolysis. The electrolytes, from which a metal may be deposited, do not yield this effect. Using other metals as negative electrodes the effect does not appear, except in the case of arsenic, when brown solid arsenic hydride is formed and floats on the surface.

S. S.

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396. *Electrical Conduction and Convection in slightly conducting Dilute Solutions.* **E. Warburg.** (Wied. Ann. 54. pp. 396–433, 1895.)—Experiments with slightly conducting liquids—as xylene, benzene—have shown that with a large fall of potential per millimetre the conduction departs from Ohm's law, whilst with a small fall per millimetre Ohm's law is satisfied. The conductivity has also been shown to increase with rise in temperature: this result does not necessarily indicate that the conduction is electrolytic. The author commences with experiments in which the current flowed between a large and small electrode in redistilled aniline. With an E.M.F. of 150 to 1200 volts a constant current is immediately obtained, which is greater if the small electrode be positive. With an E.M.F. of 8 to 16 volts there is an increase or decrease of current with time, which reoccurs whenever the direction of the current is changed. This change in value tends to a limit which is greater for the current in one direction than in the other. As an example, the ratio between the resistance for the two directions of current varies from 2·34 to 1·15 when the E.M.F. is varied from 16·8 to 1286 volts. This change with the direction of the current is reversed if xylene be mixed with the aniline, and almost disappears if a small quantity of trichloracetic acid is added to the mixture.

At the wire electrode, when the E.M.F. is large (1200 volts), convection-currents are formed; and these the author observes by two methods, one depending on the difference in refraction of the moving layers (*schlierenmethode*), and the other on the transference of lycopodium-powder scattered around the electrode. He also observes the velocity of rotation of a small electric mill moved by the convective forces. The comparison of these observations with those on resistance show that the outward-flowing convection-current is stronger at the electrode at which the current raises the specific conductivity than at the electrode at which it lowers the conductivity. This is in agreement with theory.

A tube with exploring electrodes, like that used by Hittorf for studying the distribution of potential in an exhausted gas, is used for finding the potential at various fixed points between the electrodes. It is found that the potential between the exploring electrodes varies with the time the current is maintained. In aniline the conductivity near the anode at first increases and then after many hours sinks, whilst in the centre of the tube and near the kathode the conductivity continuously falls. The effects are again reversed when xylene is added. In this tube distilled water shows large increases of conductivity, which disappear some hours after the current is stopped.

Following Kohlrausch and Heydweiller, these observations are explained by considering the substances as very dilute solutions of electrolytes. The redistribution and change in composition of the dissolved substances will account for the observed changes in conductivity. Aniline behaves as a dilute solution of KHO, and the xylene mixtures as dilute H_2SO_4. S. S.

397. *A Sensitive Galvanometer.* **P. Weiss.** (C. R. 120. pp. 728–731, 1895.)—The galvanometer described is a modification of Kelvin's astatic galvanometer. The chief change is in the construction of the astatic system, which consists of two *vertical* needles, parallel to the axis of rotation, with unlike poles as close together as is convenient. The advantages claimed are that the magnetisation is very constant, that the astaticity is easily realisable, depending only on the parallelism of the magnets to the axis of rotation, and that, for the same sensibility as with the ordinary device, the magnets are heavier and not so susceptible to slight mechanical disturbances. Details of two galvanometers are given. In one the needles are 0·2 mm. in diameter, 18 mm. long, and are placed at a distance apart equal to 1·2 mm. The mirror weighs 8 mg. and is rectangular in shape, 7 mm. by 2 mm., the longer sides being parallel to the axis of suspension. If the sensibility be measured by dividing the deflection produced by one microampere by the square root of the resistance, its value is 1200. A. Gs.

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398. *Aperiodic Electrometer, free from Magnets and Elastic Fatigue.* **W. Hallwachs.** (Nachr. Ges. Wiss. Göttingen, pp. 122–134, 1895.)—The paper contains a clear account of a carefully constructed and somewhat complicated quadrant-electrometer, along with experiments made to test its accuracy and behaviour. A platinum wire, drawn enclosed within a double covering of silver and copper, suspends the needle; its elastic fatigue is very small and produces an error of only a few ten-thousandths, under the conditiens explained. The damping is caused by a vane placed within a metal box; the needle, vane, and box are all metallically connected, and at the same potential. A. Gs.

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399. *Absolute Electrometer for High Potentials.* **H. Abraham** and **J. Lemoine.** (C. R. 120. pp. 726–728, 1895).—Two electrometers are described—one a standard, carefully constructed, with an accuracy of one part in a thousand, which reads to over 40,000 volts; the other of simpler construction, which has an accuracy of one part in a hundred and reads to 100,000 volts. Both are made on the same principle as Kelvin's guard-ring electrometer. A. Gs.

CHEMICAL PHYSICS.

400. *Influence of Temperature on the State of Dissociation of Acids, from the Experiments of E. Schröder.* **H. Jahn.** (Zschr. phys. Chem. 16. pp. 72–90, 1895.)—Measurements of the conductivity of solutions of different strengths of the acids of the acetic series were made by Kohlrausch's method, and the values of the molecular conductivity tabulated for different temperatures. Interpolation formulæ of the form

$$\lambda = A + Bt - Ct^2$$

are given for each case, and shown to represent the results with accuracy. In order to determine the limiting values of the conductivity at great dilution, determinations were made with dilute solutions of the sodium salts of the acids, of sodium chloride, and of hydrochloric acid. Since all these solutions are much more dissociated than the organic acids, they reach their limiting values at moderate dilutions. The limiting value of the conductivity of a solution of the acid was obtained from the formula

$$\lambda_\infty = \lambda_{HCl} - \lambda_{NaCl} + \lambda_{NaR}.$$

The dissociation-constant can then be calculated for each case from Ostwald's expression

$$\frac{(\lambda_v/\lambda_\infty)^2}{(1-\lambda_v/\lambda_\infty)v} = \text{constant}.$$

Except for concentrated solutions, the dissociation-constant of most of the acids came out almost independent of the temperature. In the case of valeric acid, it decreased as the temperature rose. The following is a summary of the results for the mean values of the constant :—

	At 10°.	At 20°.	At 30°.	At 40°.
Acetic acid	0·00179	0·00185	0·00187	0·00187
Propionic acid ..	0·00138	0·00142	0·00143	0·00141
Butyric acid	0·00166	0·00169	0·00168	0·00162
Isobutyric acid ..	0·00159	0·00162	0·00161	0·00156
Valeric acid	0·00184	0·00177	0·00167	0·00155

From this it follows that the heat of dissociation of such acids must be very small. The author deduces an expression for this heat of dissociation Q,

$$\frac{Q}{T^2} = \frac{2\lambda_\infty - \lambda_v}{\lambda_\infty - \lambda_v}\left(\frac{1}{\lambda_\infty}\frac{\partial\lambda_\infty}{\partial T} - \frac{1}{\lambda_v}\frac{\partial\lambda_v}{\partial T}\right),$$

which is identical with one obtained by Arrhenius by another method. W. C. D. W.

---

401. *Velocities of the Ions.* **A. Campetti.** (N. Cim. 4. 1, pp. 73–76, Feb. 1895.)—The experiments of Völlmer (Abstr. Jan. 1895, p. 37) indicate that the ratio of the limiting values of

the molecular conductivities of various salts in the three solvents, water, methyl alcohol, and ethyl alcohol, was almost the same for different salts. If this is so, the velocities of the ions must have constant ratios in these solvents, and the migration-constant of any salt must be independent of the solvent. The author has found the following values for the migration-constants :—

	Water.	Ethyl Alcohol.	Methyl Alcohol.
LiCl	0·705	0·71	0·64
$AgNO_3$	0·518	0·51	0·47

Kavalhi has found that the ratio of the diffusion-constants of various salts in water and ethyl alcohol was about equal to the ratio of the molecular conductivities, and in no case differed much from the value 3. The author has (Atti dell' Accademia di Torino, vol. 29, 1894) applied Planck's formula to calculate the difference of potential between an aqueous and an alcoholic solution of a salt. W. C. D. W.

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402. *Solubility and Electrolytic Dissociation.* **A. A. Noyes** and **C. G. Abbot.** (Zschr. phys. Chem. 16. pp. 125–138, 1895.)—From the law of mass action Nernst (Zschr. phys. Chem. 4. p. 378) has deduced two principles which control the solution of a salt in water saturated with another salt: the quantity of undissociated salt with which the solution is saturated remains constant when another salt is added; and the product of the numbers of opposite ions remains constant as well. In order to examine the truth of these principles, the authors took two salts, both in saturated solution, and investigated the influence of each on the solubility of the other. The dissociation could then be calculated. Another pair of salts containing the same ion was then used to give another value for the same dissociation, and thus the principle could be tested. If m_0 and m_0' denote the solubilities of two salts in pure water, m and m' their solubilities when both salts are present, and a_0, a_0', a, and a' the corresponding values of the dissociation, the first principle gives

$$m_0(1-a_0) = m(1-a),$$
$$m_0'(1-a_0') = m'(1-a'),$$

and the second principle (of the constancy of the products of the number of ions) gives

$$m_0^2 a_0^2 = ma(ma+m'a'),$$
$$m_0'^2 a_0'^2 = m'a'(ma+m'a').$$

From these equations the four unknown dissociation-values can be calculated. The salts used must have not very different solubilities, and be capable of analytical estimation in presence of each other. The chloride, sulphocyanate, and bromate of thallium were used, their solubilities being 0·025, 0·028, and 0·022 normal respectively. The result of the experiments was to confirm the two principles mentioned above, and to show that the dissociation

calculated from the solubilities agreed with that deduced from the electrical conductivities.

Salt.	Dissociation from the Solubilities.	Dissociation from the Conductivities.
TlCl.....	86·5±0·3	86·6±0·1
TlSCN.....	86·6±0·3	85 6±0·1
$TlBrO_3$	90·2±0·4	89·0±0·1

This confirmation of the value of the dissociation obtained from the conductivities disproves the former explanation of certain discrepancies in the law of mass action as applied to the dilution of salt solutions. These discrepancies cannot be due, as was supposed, to inaccuracies in determining the amount of dissociation. The product of the number of ions does not bear, at different dilutions, a constant ratio to the amount of undissociated salt; the product, however, for each value of the quantity of undissociated salt, is independent of the relative number of the two ions. W. C. D. W.

403. *Note on the Volumes of Salts in Aqueous Solutions.* **L. de Boisbaudran.** (C. R. 120. pp. 539–541, 1895.)—All salts, except those of ammonia, show a diminution in the total volume, when dissolved in water. But, with supersaturated solutions, the author finds that, in all cases except that of $Na_2SO_4 . 10$ aq., a contraction occurs on crystallisation, and, consequently, an expansion on solution. Crystals of such salts can be made to form at the top of the liquid by dissolving in it an auxiliary substance, which does not act chemically on the first, and is not isomorphous with it, in order to increase the density of the liquid. W. C. D. W.

404. *Cryoscopic Researches.* **A. A. Noyes** and **W. R. Whitney.** (Technol. Quarterly, 7. April 1894.)—The authors examine the freezing-points of solutions of aluminates and borates of alkali metals, and conclude that the alkaline aluminates have, in solution, the composition $MAlO_2$. The borates have a corresponding composition, as long as the number of boric acid molecules does not exceed that of the alkali molecules. The composition of the salt formed in presence of two boric acid molecules is $M_2B_4O_7$. The addition of more acid gives rise to still more complex di-sodium or di-potassium salts. W. C. D. W.

405. *Classification of the Elements.* **J. Traube.** (Zschr. an. Chem. 8. pp. 77–80, 1895.)—In Mendelejeff's system mercury is associated with zinc and cadmium instead of with copper; sodium with silver, copper, and gold, whereas it would more naturally take its place with lithium and the other alkali metals. An element generally has points of resemblance to several other elements

belonging to different groups; but the periodic system assigns to each element one particular position. After pointing out these and other objections to the periodic system, the author urges that the properties of the elements are to be regarded as functions of their atomic volumes, as well as of their atomic weights, and suggests the following arrangement of the elements in natural families :—

Fl——Cl—Br—I. Cl, Br; I. Cl, Mn.
O ——S—Se—Te. S, Cr, Se, Te, Mo, W.
N ——P—As—Sb. N, Vd. B.
C, Si—Ti—Zr—Sn.
H; Zi, Na——K——Rb——Cs.
m-Cu, Ag, *m*-Hg, *m*-Au. *m*-Tl. NH_4.
Be. *d*-Pb; Ca, Sr; Ba.
Mg, Zn; Cd.
d-Cu, Ni. *d*-Mn, *d*-Fe.
Co.
Al, *tr*-Fe, *tr*-Cr.
Pd, Pt, Ir.
Mo, W.

The appearance of an element in more than one group is generally associated with a change in valency. In order to indicate this the prefixes *mono*, *di*, *tri* (*m*-, *d*-, *tr*-) are used. D. E. J.

406. *Melting-Point of Mixtures of Isomorphous Salts.* **F. W. Küster.** (Zschr. phys. Chem. 15. pp. 86–88, 1894.)—A criticism on a paper by Le Chatelier (Compt. Rend. 118. p. 350. 415, p. 88, 1894). W. C. D. W.

407. *Simple Experiment showing the Presence of Argon in Air.* —**Guntz.** (C. R. 120. pp. 777–778, 1895.)—The author describes the following method of preparing argon from atmospheric nitrogen. Place an iron boat filled with lithium (prepared electrolytically, C. R. 117. p. 732) in a glass tube fitted with a manometer, and containing atmospheric nitrogen. When the lithium is heated to dull redness, it combines with the nitrogen with incandescence, and a vacuum is produced of about 10 mm. of mercury. Let the process be repeated, and a vacuum of about 20 mm. is obtained. The experiment may be repeated until a sufficient quantity of argon is obtained. To show that the residue is argon and not due to an incomplete absorption of nitrogen, an experiment can be made with chemically prepared nitrogen. A vacuum much better than 10 millimetres is obtained at once, and remains when the experiment is repeated. S. S.

408. *Critical Temperature as Test of Purity.* **M. Altschul.** (V. phys. Ges. Berlin, 14. pp. 1–16, 1895.) A popular lecture on subjects already noticed in these Abstracts, *see* 208, 209, 210, 238. S. S.

409. *The Molecular Deviation or the Molecular Rotative Power of Active Substances.* **A. Aignan.** (C. R. 120. pp. 733–725, 1895.)—Ph. A. Guye in Comptes Rendus, 116. 1893, makes use of the expression *molecular deviation*, which is defined by the equation

$$(\delta) = \frac{\alpha}{l}\sqrt[3]{\frac{M}{d}},$$

where α is the deviation of the plane of polarisation produced by length l of the active substance, M is the molecular weight, and d the density of the substance at the temperature of observation. He thinks that (δ) represents the deviations produced by liquid columns containing the same number of molecules. The author states that the formula used by Guye is unsuitable for the purpose intended: his reasons are, (1) because experiment shows that (δ) varies with the degree of dilution of a solution; (2) because Guye is incorrect, theoretically, in thinking that (δ) represents the deviations produced by liquid columns containing the same number of molecules. The author prefers on practical and theoretical grounds the constant, designed by himself, called the molecular rotative power, defined by the equation $[\alpha'] = M\alpha/ld$.

A. Gs.

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